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Some Fractional Calculus Operators Associated with Generalized Mittag-Leffler Funcitons

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Abstract: In this paper we have evaluated some new results with Saigo fractional integral operator first and second kinds involving generalized Mittag-Leffler functions and also evaluate the new theorem with special cases.

Keywords: Saigo Integral operators; Mittag-Leffler functions.

1. Introduction

The special functions of the form

$$E_\alpha(z)=\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n+1)}z^n(z,\alpha\in\mathbb{C},\mathbf{Re}(\alpha)>0) \tag{1}$$

and

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n+\beta)}z^n(z,\alpha,\beta\in\mathbb{C},\mathbf{Re}(\alpha)>0,\mathbf{Re}(\beta)>0) \tag{2}$$

with $\mathbb{C}$ being the set of complex number, are known as the function $E_\alpha(z)$ was introduced by Mittag-Leffler in the year of 1903. It is direct generalization of the exponential series. For $\alpha=1$ we have the exponential series. and $E_{\alpha,\beta}(z)$ was introduced by Wiman in 1905.

The main results in the classical theory of these functions can be found in the handbook by Erdelyi et al [1953] [2] and more results are given in the books by Dzherbashyan [1966],[1993] [11].

Prabhakar [1971][8] introduced the function $E^{\gamma}_{\alpha,\beta}(z)$ in the form:

$$E^{\gamma}_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)n!}z^n(z,\alpha,\beta,\gamma\in\mathbb{C},\mathbf{Re}(\alpha)>0,\mathbf{Re}(\beta)>0) \tag{3}$$

where $(\gamma)_n$ is the Pochhammer symbol, defined as:

$$(\gamma)_n=\frac{\Gamma(\gamma+n)}{\Gamma(\gamma)}=\begin{cases}1, & \text{if } n=0,\ \gamma\in\mathbb{C}\setminus\{0\},\\ \gamma(\gamma+1)(\gamma+2)\cdots(\gamma+n-1), & \text{if } n\in\mathbb{N},\ \gamma\in\mathbb{C}.\end{cases} \tag{4}$$

where $\mathbb{N}=1,2,3,...$ is the set of natural numbers and $\mathbb{N}_0=\mathbb{N}\cup\{0\}$, The function $E^{\gamma}_{\alpha,\beta}(z)$ is generalization of the exponential function $exp(z)$, Mittag- Leffler function $E_{\alpha}(z)$ and Wilman's function $E^{\gamma}_{\alpha,\beta}(z)$.[3], [9] and Kilbas and Saigo [10], [6] investigated several properties and applications of the functions defined in (1 to 3).

As Salim in [9] introduced a further generalization of the Mittag- Leffler function in the form

$$E^{\gamma,\delta}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n}{\Gamma(\alpha n+\beta)(\delta)_n} z^n \tag{5}$$

where

$$(z,\alpha,\beta,\gamma,\delta \in \mathbb{C}, \min(\mathbf{Re}(\alpha)>0, \mathbf{Re}(\beta), \mathbf{Re}(\delta))>0) \tag{6}$$

$E^{\gamma,\delta}_{\alpha,\beta}(z)$ contains the aforementioned Mittag-Leffler function.

Note that $E^{\gamma,1}_{\alpha,\beta}(z) = E^{\gamma}_{\alpha,\beta}, E^{1,1}_{\alpha,\beta}(z) = E_{\alpha,\beta}$ *and* $E^{1,1}_{\alpha,1}(z) = E_{\alpha}(z)$. Throughout this investigation, we shall use these facts to study the various properties and relations of the function $E^{\gamma,\delta}_{\alpha,\beta}(z)$

Here in this sections we have studied the fractional integral operator introduced by Saigo ([1978],[1979]),[10], [7] containing generalized Mittag-Leffler function defined in eq (5) in the kernel.

For e.g. as given as follows:

1. The basic Mittag-Leffler function is denoted by $E_{\alpha}(z)$ and it is defined as

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\alpha n+1)}, \quad \Re(\alpha) > 0.$$

Note that when $\alpha = 1$,

$$\sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n+1)} = \sum_{n=0}^{\infty} \frac{z^n}{n!} = e^z,$$

and hence $E_{\alpha}(z)$ is a generalization of the exponential series.

2. Prove that

$$E_{1,3}(z) = \frac{e^z - z - 1}{z^2}$$

$$E_{1,3}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n+3)} = \frac{1}{z^2} \sum_{n=0}^{\infty} \frac{z^{n+2}}{\Gamma(n+2)} = \frac{1}{z^2}(e^z - z - 1).$$

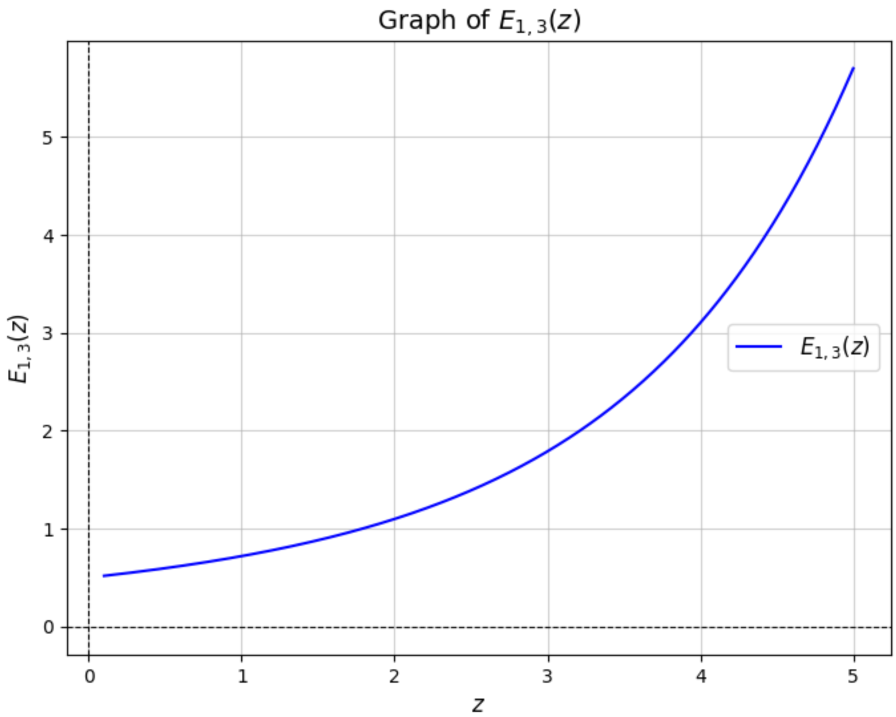

Figure 1: Numerical Graph

The plotted curve increases as z increases. This behavior suggests that $E_{1,3}(z)$ is a growing function of z, likely exponential or polynomial in nature.

3. One generalization of $E_\alpha(z)$ is denoted and defined as follows:

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(\beta+\alpha n)}, \quad \Re(\alpha) > 0, \Re(\beta) > 0.$$

This is a 2-parameter generalization of $E_\alpha(z)$.

A 3-parameter generalization of $E_\alpha(z)$ is denoted by $E^{\gamma}_{\alpha,\beta}(z)$ and it is defined as

$$E^{\gamma}_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n z^n}{n!\Gamma(\beta+\alpha n)}, \quad \Re(\alpha) > 0, \Re(\beta) > 0.$$

2. Fractional Integrals and Derivatives

The fractional integrals and derivatives which will be required are defined as follows[5], [4], [13], [12]:

Let α, β, and η be complex numbers, and let $x \in \mathbb{R}_+ = (0,\infty)$.

Following [5], [4] the fractional integral($\mathrm{Re}(\alpha) > 0$) and derivative ($\mathrm{Re}(\alpha) < 0$) of first kind of a function $f(x)$ on $\mathbb{R}_+$ are defined respectively in the forms:

$$\begin{cases} I^{\alpha,\beta,\eta}_{0,x} f = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \ {}_2F_1\left(\alpha+\beta, -\eta; \alpha; 1-\frac{t}{x}\right) f(t)dt, \mathrm{Re}(\alpha) > 0 \\ = \frac{d^n}{dx^n} I^{\alpha+n,\beta-n,\eta-n}_{0,x} f, 0 < \mathrm{Re}(\alpha)+n \leq 1, (n = 1,2,3...) \end{cases} \tag{7}$$

where ${}_2F_1(.)$ is Gauss's hypergeometric function.

Fractional integral($\mathrm{Re}(\alpha) > 0$) and derivative ($\mathrm{Re}(\alpha) < 0$) of second kind of a function $f(x)$ on $\mathbb{R}_+$ are given by:

$$\begin{cases} J^{\alpha,\beta,\eta}_{x,\infty} f = \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\beta} \ {}_2F_1\left(\alpha+\beta, -\eta; \alpha; 1-\frac{x}{t}\right) f(t)dt, \mathrm{Re}(\alpha) > 0 \\ = (-1)^n \frac{d^n}{dx^n} J^{\alpha+n,-\beta-n,\eta}_{x,\infty} f, 0 < \mathrm{Re}(\alpha)+n \leq 1, (n = 1,2,3...) \end{cases} \tag{8}$$

3. The Riemann-Liouville, Weyl and Erdelyi-Kober fractional operators

The Riemann-Liouville, Weyl and Erdelyi-Kober fractional operators are interpreted as special cases of the operators I and J:

$$\begin{cases} R_{0,x}^{\alpha} f = I_{0,x}^{\alpha,-\alpha,\eta} f = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \mathbf{Re}(\alpha) > 0 \\ = \frac{d^n}{dx^n} R_{0,x}^{\alpha+n} f, 0 < \mathbf{Re}(\alpha) + n \leq 1, (n = 1,2,3,...) \end{cases} \tag{9}$$

$$\begin{cases} W_{x,\infty}^{\alpha} f = J_{x,\infty}^{\alpha,-\alpha,\eta} f = \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} f(t)dt, \mathbf{Re}(\alpha) > 0 \\ = (-1)^n \frac{d^n}{dx^n} W_{x,\infty}^{\alpha+n} f, 0 < \mathbf{Re}(\alpha) + n \leq 1, (n = 1,2,3,...) \end{cases} \tag{10}$$

$$E_{0,x}^{\alpha,\eta} f = I_{0,x}^{\alpha,0,\eta} f = \frac{1}{\Gamma(\alpha)} x^{-\alpha-\eta} \int_0^x (x-t)^{\alpha-1} t^{\eta} f(t)dt, \mathbf{Re}(\alpha) > 0 \tag{11}$$

and

$$k_{x,\infty}^{\alpha,\eta} f = J_{x,\infty}^{\alpha,0,\eta} f = \frac{1}{\Gamma(\alpha)} x^{\eta} \int_0^x (t-x)^{\alpha-1} t^{-\alpha-\eta} f(t)dt, \mathbf{Re}(\alpha) > 0 \tag{12}$$

In this section we have made use of right-sided Riemann-Liouville fractional integral operators $I_{a^+}^{P}$ and the right-sided Riemann-Liouville fractional derivative operator $D_{a^+}^{P}$ which are studied by [6], [13], [12] and defined as

$$(I_{a^+}^{\mu} f)(x) = \frac{1}{\Gamma(\mu)} \int_a^x \frac{f(t)}{(x-t)^{1-\mu}} dt, R(\mu) > 0 \tag{13}$$

and

$$(D_{a^+}^{\mu} f)(x) = \left(\frac{d}{dx}\right)^n (I_{a^+}^{n-\mu} f)(x), R(\mu) > 0; n = [R(\mu)] + 1 \tag{14}$$

Where$[x]$ denote the greatest integer in the real number x. We remark in passing that Hilter [2000]. [2006] generalized Riemann-Liouville fractional derivative operator $D_{a^+}^{\mu}$ in (14)by

introducing a right-sided fractional derivative operator $D_{a^+}^{\mu;\nu}$ of order $0 < \mu < 1$ and type $0 \leq \nu \leq 1$ with respect to x as follows:

$$(D_{a^+}^{\mu;\nu} f)(x) = \left(I_{a^+}^{\nu(1-\mu)} \frac{d}{dx} \left(I_{a^+}^{(1-\nu)(1-\mu)} f\right)\right)(x) \tag{15}$$

The generalization (15) yields the classical Riemann-Liouville fractional derivative operator $D_{a^+}^{\mu}$ when $\nu = 0$. Moreover, in its special case when $\nu = 1$, the defination (15) would reduce to the familiar Caputo fractional derivative operators in [4].

4. Saigo Fractional Integrals Involving Generalized Mittag-Leffler Function

Theorem 1: Here in this section, we have obtained the fractional integral of the first kind involving the generalized Mittag-Leffler function:

$$\left(I_{0,x}^{\alpha,\beta,\eta}\left[t^{\rho-1} E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})\right]\right)(x) = x^{-\beta+\rho-1} \frac{\Gamma(\delta)}{\Gamma(\gamma)} \sum_{n=0}^{\infty} \frac{(\alpha+\beta)_n(-\eta)_n}{n!} {}_3\psi_3 \left[\begin{matrix} (\gamma,1),(1,1),(\rho,\beta); \\ (\delta,1),(\beta,\alpha),(\alpha+\rho+n,\beta); \end{matrix} (ax^{\beta})\right] \tag{16}$$

provided $\mathbf{Re}(x) > 0, \mathbf{Re}(\gamma) > 0$ and fractional integral of second kind involving Generalized Mittag-Leffler function as:

$$\left(J_{x,\infty}^{\alpha,\beta,\eta}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{-\beta})\right]\right)(x) = x^{-\beta+\rho-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{(\alpha+\beta)_n(-\eta)_n}{n!}\,{}_3\psi_3 \left[\begin{matrix}(\gamma,1),(1,1),(1+\beta-\rho,\beta); \\ (\delta,1),(\beta,\alpha),(1+\alpha+\beta-\rho+n,\beta);\end{matrix}\; (ax^{-\beta})\right], \tag{17}$$

provided $\mathbf{Re}(x) > 0, \mathbf{Re}(\gamma) > 0$,

$$(\alpha,\beta,\gamma,\delta \in \mathbb{C}, \min(\mathbf{Re}(\alpha) > 0, \mathbf{Re}(\beta), \mathbf{Re}(\delta)) > 0)$$

Proof: Prof for (16): Let us take the left hand side as

$$\left(I_{0,x}^{\alpha,\beta,\eta}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})\right]\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}\;{}_2F_1\left(\alpha+\beta,-\eta;\alpha;1-\frac{t}{x}\right)t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})dt$$
$$= \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)}\int_0^x (x-t)^{\alpha-1}\sum_{n=0}^{\infty}\frac{(\alpha+\beta)_n(-\eta)_n}{(\alpha)_n n! x^n}(x-t)^n t^{\rho-1}\sum_{n=0}^{\infty}\frac{(\gamma)_n a^n t^{n\beta}}{\Gamma(\alpha n+\beta)(\delta)_n}dt \tag{18}$$

putting $\frac{t}{x} = u$ and interchanging the order of integration and summation which is permissible under the conditions stated and then using the beta integral appropriately, we get

$$x^{\rho-\beta-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{(\alpha+\beta)_n(-\eta)_n}{n!}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(\rho,\beta); \\ (\delta,1),(\beta,\alpha),(\alpha+\rho+n,\beta);\end{matrix}\; ax^{\beta}\right] \tag{19}$$

which is the requirement result(16)
Similarly we can prove the following results for the Riemann-Liouville, Weyl and Erdelyi-Kober fractional operators involving generalized Mittag-Leffler function.

$$\left(I_{0^+}^{\alpha}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})\right]\right)(x) = x^{\alpha+\rho-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(\rho,\beta); \\ (\delta,1),(\beta,\alpha),(\alpha+\rho,\beta);\end{matrix}\; ax^{\beta}\right] \tag{20}$$

$$\left(D_{0^+}^{\alpha}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})\right]\right)(x) = x^{-\alpha+\rho-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(\rho,\beta); \\ (\delta,1),(\beta,\alpha),(-\alpha+\rho,\beta);\end{matrix}\; ax^{\beta}\right] \tag{21}$$

$$\left(I_{-}^{\alpha}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})\right]\right)(x) = x^{-\rho}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(\rho,\beta); \\ (\delta,1),(\beta,\alpha),(\alpha+\rho,\beta);\end{matrix}\; ax^{-\beta}\right] \tag{22}$$

$$\left(D_{-}^{\alpha}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{-\beta})\right]\right)(x) = x^{-\rho}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(\rho,\beta); \\ (\delta,1),(\beta,\alpha),(-\alpha+\rho,-\beta);\end{matrix}\; ax^{\beta}\right] \tag{23}$$

$$\left(W_{x,\infty}^{\alpha}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})\right]\right)(x) = x^{\alpha+\rho-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(1-\alpha-\rho,\beta); \\ (\delta,1),(\beta,\alpha),(1-\alpha-\beta,\beta);\end{matrix}\; ax^{-\beta}\right] \tag{24}$$

$$\left(E_{0,x}^{\alpha,\eta}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{\beta})\right]\right)(x) = x^{\rho-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(\eta+\rho,\beta); \\ (\delta,1),(\beta,\alpha),(\alpha+\rho+\eta,\beta);\end{matrix}\; ax^{\beta}\right] \tag{25}$$

$$\left(K_{x,\infty}^{-\alpha,\eta}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma,\delta}(at^{-\beta})\right]\right)(x)=x^{\rho-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_3\psi_3\left[\begin{matrix}(\gamma,1),(1,1),(1+\eta-\rho,\beta);\\(\delta,1),(\beta,\alpha),(1+\alpha-\rho+\eta,\beta);\end{matrix}\;ax^{-\beta}\right] \tag{26}$$

Special Cases:
(i) Putting $\delta=1$ in (16), we have

$$\left(I_{0,x}^{\alpha,\beta,\eta}\left[t^{\rho-1}E_{\alpha,\beta}^{\gamma}(at^{\beta})\right]\right)(x)=\frac{x^{-\beta+\rho-1}}{\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{(\alpha+\beta)_n(-\eta)_n}{n!}\,{}_2\psi_2\left[\begin{matrix}(\gamma,1),(\rho,\beta);\\(\beta,\alpha),(\alpha+\rho+n,\beta);\end{matrix}\;(ax^{\beta})\right] \tag{27}$$

(ii) Putting $\delta=\gamma=1$ in (16), we have

$$\left(I_{0,x}^{\alpha,\beta,\eta}\left[t^{\rho-1}E_{\alpha,\beta}(at^{\beta})\right]\right)(x)=x^{-\beta+\rho-1}\sum_{n=0}^{\infty}\frac{(\alpha+\beta)_n(-\eta)_n}{n!}\,{}_2\psi_2\left[\begin{matrix}(\rho,\beta),(1,1);\\(\beta,\alpha),(\alpha+\rho+n,\beta);\end{matrix}\;(ax^{\beta})\right] \tag{28}$$

(iii) Putting $\delta=\gamma=\beta=1$ in (16), we have

$$\left(I_{0,x}^{\alpha,\beta,\eta}\left[t^{\rho-1}E_{\alpha}(at^{\beta})\right]\right)(x)=x^{-\beta+\rho-1}\sum_{n=0}^{\infty}\frac{(\alpha+\beta)_k(-\eta)_n}{n!}\,{}_2\psi_2\left[\begin{matrix}(1,1),(\rho,\beta);\\(1,\alpha),(\alpha+\rho+n,\beta);\end{matrix}\;(ax^{\beta})\right] \tag{29}$$

Theorem 2: Let $x>a\,(a\in\mathbb{R}^+=[0,\infty))\,,0<\mu<1,0\leq\nu\leq1$ and $\mathbf{Re}(\alpha)>\max\{0,\mathbf{Re}(\delta)-1\}$, $\min\{\mathbf{Re}(\beta),\mathbf{Re}(\delta),\mathbf{Re}(\lambda)\}>0$ and $\gamma,\omega\in\mathbb{C}$, then

$$\left(I_{a^+}^{\lambda}\left[(t-a)^{\beta-1}E_{\alpha,\beta}^{\gamma,\delta}\left[\omega(t-a)^{\alpha}\right]\right]\right)(x)=(x-a)^{\beta+\lambda-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_2\psi_2\left[\begin{matrix}(\gamma,1),(1,1);\\(\delta,1),(\lambda+\beta,\alpha);\end{matrix}\;\omega(x-a)^{\alpha}\right] \tag{30}$$

Proof: Let

$$\left(D_{a^+}^{\lambda}\left[(t-a)^{\beta-1}E_{\alpha,\beta}^{\gamma,\delta}\left[\omega(t-a)^{\alpha}\right]\right]\right)(x)=(x-a)^{\beta-\lambda-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\,{}_2\psi_2\left[\begin{matrix}(\gamma,1),(1,1);\\(\delta,1),(-\lambda+\beta,\alpha);\end{matrix}\;\omega(x-a)^{\alpha}\right] \tag{31}$$

and

$$\left(D_{a^+}^{\mu,\nu}\left[(t-a)^{\beta-1}E_{\alpha,\beta}^{\gamma,\delta}\left[\omega(t-a)^{\alpha}\right]\right]\right)(x)=(x-a)^{\beta-\lambda-1}\,{}_2\psi_2\left[\begin{matrix}(\gamma,1),(1,1);\\(\delta,1),(\beta-\mu,\alpha);\end{matrix}\;\omega(x-a)^{\alpha}\right] \tag{32}$$

Proof of (30) : using the defination(27), the left hand side can be written as

$$\begin{aligned}&\left(I_{a^+}^{\lambda}\left[(t-a)^{\beta-1}E_{\alpha,\beta}^{\gamma,\delta}\left[\omega(t-a)^{\alpha}\right]\right]\right)(x)\\ &=\frac{1}{\Gamma(\lambda)}\int_a^x(x-t)^{\lambda-1}(t-a)^{\beta-1}E_{\alpha,\beta}^{\gamma,\delta}[\omega(t-a)^{\alpha}]dt\end{aligned} \tag{33}$$

Expressing $E_{\alpha,\beta}^{\gamma,\delta}$ by its series expansion formula, we have

$$\frac{1}{\Gamma(\lambda)}\int_a^x(x-t)^{\lambda-1}(t-a)^{\beta-1}\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)(\delta)_n}\omega^n(t-a)^{\alpha n}dt \tag{34}$$

Interchanging the order of integration and summation which is permission under the conditions stated and then using the beta integral appropriately, we get

$$(x-a)^{\beta+\lambda-1}\frac{\Gamma(\delta)}{\Gamma(\gamma)}\ {}_2\psi_2\left[\begin{matrix}(\gamma,1),(1,1); & \\ & \omega(x-a)^{\alpha}\\ (\delta,1),(\beta+\lambda,\alpha); & \end{matrix}\right] \tag{35}$$

which is the required result.
Special Cases:
(i) Putting $\delta=1$ in (30) we have

$$\begin{aligned}&\left(I_{a^+}^{\lambda}\left[(t-a)^{\beta-1}E_{\alpha,\beta}^{\gamma}\left[\omega(t-a)^{\alpha}\right]\right]\right)(x)=\\ &\frac{(x-a)^{\beta+\lambda-1}}{\Gamma(\gamma)}\ {}_1\psi_1\left[\begin{matrix}(\gamma,1); & \\ & \omega(x-a)^{\alpha}\\ (\beta+\lambda,\alpha); & \end{matrix}\right]\end{aligned} \tag{36}$$

(ii) Putting $\delta=1$and $\gamma=1$ in(30) we have

$$\begin{aligned}&\left(I_{a^+}^{\lambda}\left[(t-a)^{\beta-1}E_{\alpha,\beta}\left[\omega(t-a)^{\alpha}\right]\right]\right)(x)=\\ &\frac{(x-a)^{\beta+\lambda-1}}{\Gamma(\gamma)}\ {}_1\psi_1\left[\begin{matrix}(1,1); & \\ & \omega(x-a)^{\alpha}\\ (\beta+\lambda,\alpha); & \end{matrix}\right]\end{aligned} \tag{37}$$

(iii) Putting $\delta=1$ and $\gamma=\beta=1$in (30) we have

$$\begin{aligned}&\left(I_{a^+}^{\lambda}\left[(t-a)^{\beta-1}E_{\alpha}\left[\omega(t-a)^{\alpha}\right]\right]\right)(x)=\\ &\frac{(x-a)^{\beta+\lambda-1}}{\Gamma(\gamma)}\ {}_2\psi_2\left[\begin{matrix}(\beta,\alpha),(1,1); & \\ & \omega(x-a)^{\alpha}\\ (1,\alpha),(\beta+\lambda,\alpha); & \end{matrix}\right]\end{aligned} \tag{38}$$

5. Application

The application of Saigo integral operators and Mittag-Leffler functions provides a powerful framework for solving complex problems in fractional calculus. These tools are particularly useful in modeling phenomena such as anomalous diffusion, where the Mittag-Leffler function generalizes exponential decay to describe processes with memory effects. For example, in heat conduction in fractal media, the Saigo operator can be used to derive solutions that better capture the non-local behavior of the system compared to classical approaches.

6. Conclusion

The study explores the application of Saigo integral operators in conjunction with Mittag-Leffler functions, deriving significant results and establishing new theorems that include various special cases, thereby advancing the understanding of their interplay in fractional calculus.

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