Journal of Mathematical Modeling and Fractional Calculus
https://acspublisher.com/journals/index.php/jmmfc
Received:16-06-2025, **Revised:** 17-08-2025
Accepted:18-12-2025, **Published:** 30-12-2025
https://doi.org/10.48165/jmmfc.2025.2201, 2(2),1-14.

Dimensional Analysis of the Tickysim Spiking Neural Network: Insights and Applications

Umar Farooq[a], Faryal Chaudhry[a], Wasim Abbas[a]

[a]Department of Mathematics and Statistics, The University of Lahore, 54000, Pakistan

Corresponding author Email: 70126471@student.uol.edu.pk
dr.umar.farooq75@gmail.com

Abstract: Graph theory is like the backbone of our understanding of complex networks, whether they be social, computational, or biological. In this study, we dive into the fascinating world of the Tickysim Spiking Neural Network (TSNN) to explore its metric, edge metric, and fault-tolerant metric dimensions. We aim To unravel its structure's intricacies and uncover its potential applications. At its core, the metric dimension tells us how many vertices we need to pinpoint the locations of all other vertices based solely on distance measurements. Similarly, the edge metric dimension reveals the minimal number of edges necessary to achieve the same goal. To add another layer of reliability, the fault-tolerant metric dimension ensures that the network can still be recognized even when some vertices or edges are out of action. Throughout our research, we discovered unique symmetries and structural features of TSNN, which enabled us to identify resolving sets for both vertex- and edge-based metrics. What's exciting is that these resolving sets work consistently, regardless of how we label the network, allowing for reliable identification even in complex scenarios. Notably, we found that the fault-tolerant metric dimension of TSNN is 3, while its metric dimension stands at 2, highlighting the network's impressive adaptability and resilience. By exploring these dimensions, our work sheds new light on the reliability and flexibility of TSNN, emphasizing its potential for groundbreaking advancements in areas like computational neuroscience and neural network modeling. We believe these insights not only enrich the theoretical landscape of graph theory but also pave the way for innovative applications in fields that thrive on robust and intricate network designs.

Keywords: Metric Dimension, Edge Metric Dimension, Fault-tolerant metric dimension, Tickysim Spiking Neural Network, Resolving Sets

1. Introduction

In the modern, quickly changing digital world, cryptography acts as a decisive hurdle for the safekeeping of confidential data and the security of communication channels against more and more sophisticated cyber threats. Along with steady updates, this governing area uses math and network theories at its core to build up defenses. In the area of graph theory, especially, the metric dimension being one of the actors that offers some of the most practical tools to enable cybersecurity and speed up technological progress is one unresolved area that continues to dwindle [1, 2, 3]. The connection between graph theory and chemistry, which is known also as mathematical chemistry, uses multidimensional solutions to chemical problems and gives mathematicians practical situations that allow them to create new formulas and techniques for applications in the world. This interdisciplinary collaboration accelerates mathematical developments and increases efficiency, ultimately reducing costs associated with chemical experimentation and personnel [4, 5].

Moreover, graph theory extends its boundaries beyond chemistry into various scientific and industrial domains, including medicine, industry, networking, artificial intelligence, and more. Applications range from designing circuits in electronics, planning routes in transportation networks, optimizing schedules in education, to enhancing strategies in military operations. These diverse applications underscore the broad relevance and importance of graph-theoretical concepts in addressing complex challenges across different fields [6, 7, 8].

2. Metric Dimension

2.1. Definition and Concept

Metric dimension is a fundamental parameter in graph theory that determines the minimum number of vertices required to uniquely identify all other vertices in a graph based on their pairwise distances. Formally, for a graph $G = (V, E)$, the metric dimension $\mu(G)$ is defined as the smallest cardinality of a vertex subset $S \subseteq V$ such that every vertex in V can be uniquely determined by its distances to the vertices in S [9, 10].To see its importance, we have presented data about the metric dimension concept in different science areas (see Figure 1).

Field	Percentage of Papers
Mathematics	42.5%
Computer Science	25.3%
Engineering	8.4%
Physics	6.4%
Decision Sciences	3.1%
Material Science	2.3%

Table 1: Distribution of Papers Across Fields (Scopus Database)

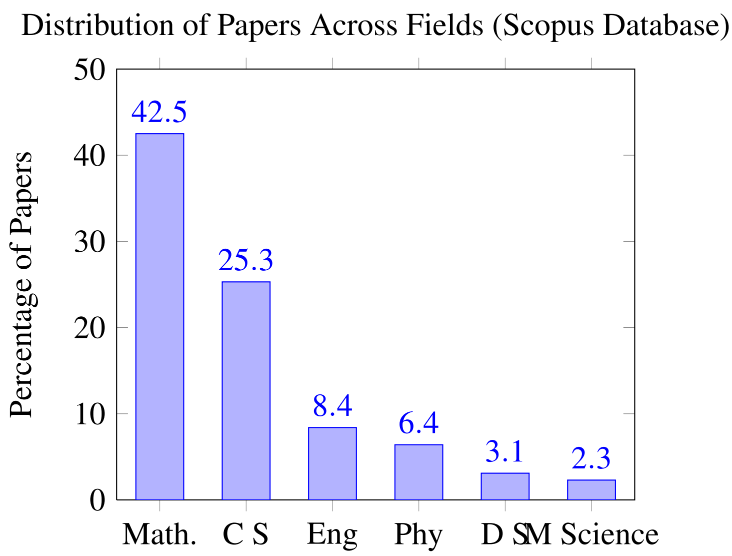
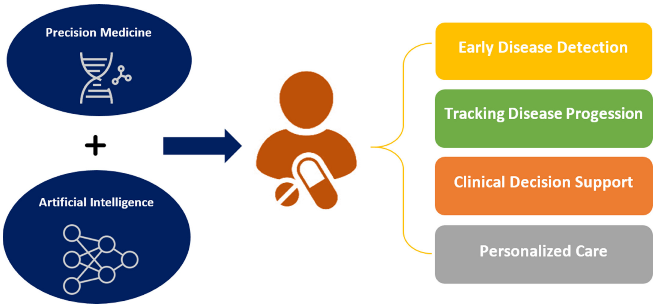


Figure 1: Bar Graph of Paper Distribution Across Fields

The metric dimension and IA work together for diagnosing different forms of deases and clarifying the stages of desease as shown in the blew figure .



2.jpg

Figure 2: Transformative integration of AI in precision medicine and MD. [11]

Now, we can briefly see its importance in different fields of science

2.2. Applications in Chemistry

In chemistry, the metric dimension plays a pivotal role in analyzing molecular structures and networks. By identifying a minimal set of vertices that uniquely determine the structure, metric dimension facilitates the systematic arrangement and graphical representation of complex chemical data. This approach enhances precision in experiments, improves the efficiency of detecting subtle changes in molecular configurations, and supports advancements in pharmaceutical research and materials science.

2.3. Medical and Industrial Applications

2.3.1. Medical Practice

Metric dimension contributes to precise dosage calculations in medical treatments, enabling tailored therapies based on patient-specific metrics such as weight and medical history. It also supports biomedical research by providing structured data analysis and comprehensive patient evaluations, thereby advancing personalized medicine [12].

2.3.2. Industrial Use

In industrial settings, metric dimension optimizes manufacturing processes by ensuring consistency and precision in operations. From design to production, it facilitates global standardization of products, enhances quality assurance, and improves resource management efficiency. Applications range from optimizing supply chain logistics to designing robust and reliable production systems [13].

2.4. Network Design and Reliability

Metric dimension plays a critical role in network design, particularly in wireless communication systems and distributed computing environments. By minimizing signal interference and ensuring fault-tolerant operation, metric dimension enhances network reliability and resilience against hardware failures and cyber-attacks. Applications include designing resilient communication infrastructures, optimizing data transmission efficiency, and ensuring continuous service availability in dynamic and challenging environments [14].

The integration of metric dimension into various scientific and industrial fields underscores its critical role in enhancing efficiency, reliability, and precision. By leveraging graph-theoretical concepts like metric dimension, researchers and practitioners can address complex challenges in network design, cybersecurity, medicine, and beyond, paving the way for innovative solutions and sustainable advancements in the digital era.

2.5. *Tickysim Spiking Neural Network*

Tickysim spiking neural networks represent a specialized area of study within computational neuroscience and neural network modeling. These networks simulate the behavior of neurons and synapses in biological systems, providing insights into neural information processing and network dynamics [15].

The configuration of Tickysim spiking neural networks lends itself well to the application of graph theory concepts, particularly metric dimension. These networks are created by neurons together which are the vertices and edges which signify the synaptic connections and as a result, lead to the formation of the complex graph structure [16]. The study of the metric dimension of the networks in Tickysim includes the determination of the minimum number of landmark neurons that will enable solving the problem of the network in which other neurons are provisioned at the same time. Regarding the metric space, such a function is one of the main basic concepts in general topology and common metric space theory [17, 18]. In addition, the calculation of the metric dimension is an important step in determining the topological and structural properties of the networks, and it is a key factor that facilitates the development of accurate models that can be used to simulate various processes of the neural system. The focus of this research is to examine the geometric and connectivity properties of the Tickysim spiking neural networks in order to compute their metric dimension. In this connection, the identification of the minimal resolving sets that are responsible for the unique identification of the neurons is a step. We aim to accomplish this in order to have a wider and deeper knowledge of these networks in modeling complex neural phenomena f.e. Hodgkin-Huxley-type and integrate-and-fire models of neurons [19, 20]. Along with the research, we point out the capacity of Tickysim spiking neural networks to increase technological innovations and the information about neural information processing in living bodies that are always on the lead with scientists

2.6. *Structure of Tickysim Spiking Neutral Network (TSNN)*

The Tickysim Spiking Neutral Network (TSNN) we are investigating consists of $m \times r$ vertices arranged in r rows and m columns, where r and m are equal, making it a symmetrical graph. We start with $n = 3$ as the base unit structure, as shown in Figure 4, and extend our analysis to $n = z$.

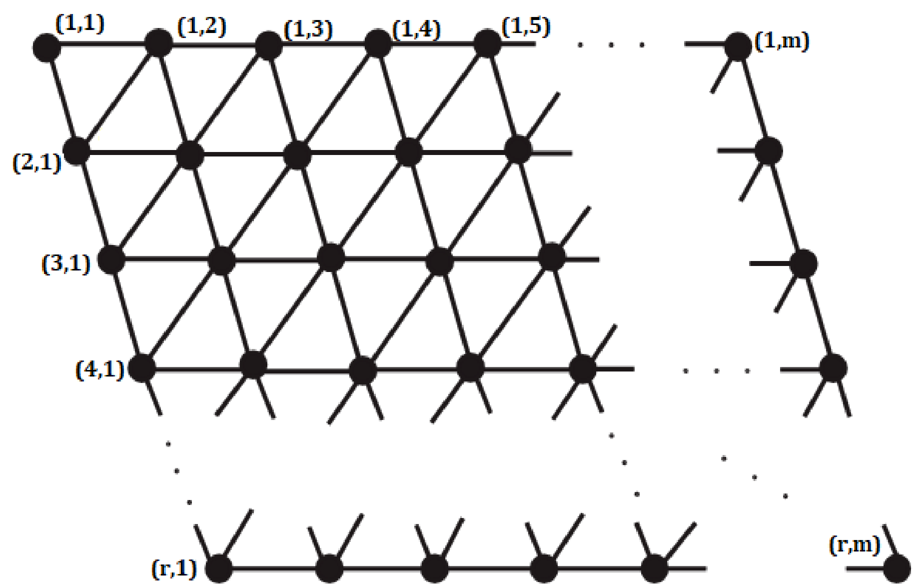


Figure 3: Structure of Tickysim Spiking Neural Network (TSNN) for $r \times m$

The vertices and edges of TSNN for various values of n are summarized in Table 2.

TSNN	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=z$
Total Vertices	4	9	16	25	36	49	$(n+1)^2$
Total Edges	5	16	33	56	85	122	$3n^2+2n$

Table 2: Vertices and edges of Tickysim Spiking Neural Network (TSNN)

3. Results Computed for TSNN

The main objective of the study will be seen in Theorem 1, and a unique subset for checking the resolvability of $V(G)$ is selected.

Theorem 3.1. *Let $TSNN$ be the graph of the Tickysim Spiking Neural Network. The metric dimension of $TSNN$ is 2, meaning there exist two vertices in $TSNN$ such that their distance vectors uniquely identify all other vertices in the graph.*

Proof. Let $W = \{a_{1,1}, a_{n+1,1}\}$ be an ordered vertex subset of $TSNN$. We will show that W is the resolving set. Below are the unique depictions of the vertices of $TSNN$ concerning W.

For $n = 1$, we have the resolving set $W = \{a_{1,1}, a_{2,1}\}$:

$$r'(a_{r_\delta,c_\xi} \mid W) = \begin{cases} (c_\xi - 1, 1) & \text{if } r_\delta = 1, c_\xi = 1,2 \\ (c_\xi, c_\xi - 1) & \text{if } r_\delta = 2, c_\xi = 1,2 \end{cases}$$

For $n = 2$, we have the resolving set $W = \{a_{1,1}, a_{3,1}\}$:

$$r'(a_{r_\delta,c_\xi} \mid W) = \begin{cases} (c_\xi - 1, 2) & \text{if } r_\delta = 1, c_\xi = 1,2,3 \\ (c_\xi, 1) & \text{if } r_\delta = 2, c_\xi = 1,2 \\ (c_\xi, 2) & \text{if } r_\delta = 2, c_\xi = 3 \\ (c_\xi + 1, c_\xi - 1) & \text{if } r_\delta = 3, c_\xi = 1,2,3 \end{cases}$$

For $n = 3$, we have the resolving set $W = \{a_{1,1}, a_{4,1}\}$:

$$r'(a_{r_\delta,c_\xi} \mid W) = \begin{cases} (c_\xi - 1, 3) & \text{if } r_\delta = 1, c_\xi = 1,2,3,4 \\ (c_\xi, 2) & \text{if } r_\delta = 2, c_\xi = 1,2,3 \\ (c_\xi, 3) & \text{if } r_\delta = 2, c_\xi = 4 \\ (c_\xi + 1, 1) & \text{if } r_\delta = 3, c_\xi = 1 \\ (c_\xi + 1, c_\xi + 1) & \text{if } r_\delta = 3, c_\xi = 2,3,4 \end{cases}$$

For $n = 4$, we have the resolving set $W = \{a_{1,1}, a_{5,1}\}$:

$$r'(a_{r_\delta,c_\xi} \mid W) = \begin{cases} (c_\xi, 4) & \text{if } r_\delta = 1, c_\xi = 1,\ldots,5 \\ (c_\xi, 3) & \text{if } r_\delta = 2, c_\xi = 1,\ldots,4 \\ (c_\xi + 1, 2) & \text{if } r_\delta = 3, c_\xi = 1,2,3 \\ (c_\xi + 1, c_\xi - 1) & \text{if } r_\delta = 4, c_\xi = 1,2,3,4 \\ (c_\xi + 3, c_\xi - 1) & \text{if } r_\delta = 5, c_\xi = 1,\ldots,5 \end{cases}$$

For $n = 5$, we have the resolving set $W = \{a_{1,1}, a_{6,1}\}$:

$$r'(a_{r_\delta,c_\xi} \mid W) = \begin{cases} (c_\xi, 5) & \text{if } r_\delta = 1, c_\xi = 1,\ldots,6 \\ (c_\xi, 4) & \text{if } r_\delta = 2, c_\xi = 1,\ldots,5 \\ (c_\xi + 1, 3) & \text{if } r_\delta = 3, c_\xi = 1,\ldots,4 \\ (c_\xi + 2, 2) & \text{if } r_\delta = 4, c_\xi = 1,\ldots,3 \\ (c_\xi + 3, 1) & \text{if } r_\delta = 5, c_\xi = 1,2 \\ (c_\xi + 3, c_\xi - 1) & \text{if } r_\delta = 5, c_\xi = 3,\ldots,6 \\ (c_\xi + 5, c_\xi - 1) & \text{if } r_\delta = 6, c_\xi = 1,\ldots,6 \end{cases}$$

For $n = 6$, we have the resolving set $W = \{a_{1,1}, a_{7,1}\}$:

$$r'(a_{r_\delta,c_\xi} \mid W) = \begin{cases} (c_\xi, 6) & \text{if } r_\delta = 1, c_\xi = 1,\dots,7 \\ (c_\xi, 5) & \text{if } r_\delta = 2, c_\xi = 1,\dots,6 \\ (c_\xi+1, 4) & \text{if } r_\delta = 3, c_\xi = 1,\dots,5 \\ (c_\xi+2, 3) & \text{if } r_\delta = 4, c_\xi = 1,\dots,4 \\ (c_\xi+3, 2) & \text{if } r_\delta = 5, c_\xi = 1,\dots,3 \\ (c_\xi+3, c_\xi-1) & \text{if } r_\delta = 6, c_\xi = 1,\dots,7 \\ (c_\xi+6, c_\xi-1) & \text{if } r_\delta = 7, c_\xi = 1,\dots,7 \end{cases}$$

For n, we have the resolving set $W = \{a_{1,1}, a_{n+1,1}\}$:

$$r'(a_{r_\delta,c_\xi} \mid W) = \begin{cases} (c_\xi, n) & \text{if } r_\delta = 1, c_\xi = 1,\dots,n+1 \\ (c_\xi, n-1) & \text{if } r_\delta = 2, c_\xi = 1,\dots,n \\ (c_\xi+1, n-2) & \text{if } r_\delta = 3, c_\xi = 1,\dots,n-1 \\ (c_\xi+2, n-3) & \text{if } r_\delta = 4, c_\xi = 1,\dots,n-2 \\ \vdots & \\ (c_\xi+(n-3), 1) & \text{if } r_\delta = n-2, c_\xi = 1,\dots,3 \\ (c_\xi+(n-2), c_\xi-1) & \text{if } r_\delta = n-1, c_\xi = 1,\dots,n \\ (c_\xi+n, c_\xi-1) & \text{if } r_\delta = n+1, c_\xi = 1,\dots,n+1 \end{cases}$$

Since the representations for W are unique, it is a resolving set with cardinality 2.

For any vertex a_{r_δ,c_ξ} in $TSNN$, we need to show that $(d(a_{r_\delta,c_\xi}, a_{n,2}), d(a_{r_\delta,c_\xi}, a_{n,3}), d(a_{r_\delta,c_\xi}, a_{n,2n+3}))$ is unique. Since the distances from a_{r_δ,c_ξ} to $a_{n,2}$, $a_{n,3}$, and $a_{n,2n+3}$ are different for different a_{r_δ,c_ξ} (as shown in the proof of the resolving set), thus the subset W is a resolving set.

□

Remarks Let $TSNN$ denote the graph of the Tickysim Spiking Neural Network. According to Theorem 1, the metric dimension of $TSNN$ is determined to be 2:

$$\dim(TSNN) = 2$$

4. Edge Metric Dimension

In graph theory, the *edge metric dimension* (EMD) is an extension of the metric dimension, focusing on the unique identification of edges within a network rather than vertices. Formally, the edge metric dimension of a graph is the minimum number of edges (known as an *edge resolving set*) needed to distinguish all other edges by their respective distances to these reference edges [21, 22].

4.1. Importance of Edge Metric Dimension

The concept of edge metric dimension is a complementary yet crucial extension to the metric dimension in network analysis. While the metric dimension enables unique identification of vertices, the edge metric dimension allows for each edge in the graph to be uniquely identified. This distinction is particularly important in applications where edge-based attributes—such as pathways, connections, or flows—are as significant as the nodes themselves [23, 24].

In networks such as the *Tickysim Spiking Neural Network*, where connections between vertices are critical to the network's overall function, the ability to uniquely identify edges provides a more comprehensive understanding of the network's structure and behavior. For example, in computational neuroscience, distinguishing edges is essential for modeling synaptic connections and understanding signal pathways, making the edge metric dimension highly relevant for analyzing and designing neural networks [25, 26].

4.2. *Why Edge Metric Dimension Matters Alongside Metric Dimension*

Although both metric dimensions contribute to understanding network structure, each offers unique insights:

- **Metric Dimension:** Primarily addresses the problem of vertex identification. It determines the minimum set of vertices required to uniquely identify every other vertex based on distance, which is crucial for understanding positional relationships within the network [27].
- **Edge Metric Dimension:** Focuses on distinguishing edges, providing a level of resolution that vertex-based identification alone cannot achieve. Even in the presence of the metric dimension, the edge metric dimension adds value by offering insights specifically about the pathways and connections. For networks where interactions (edges) are essential such as in transportation systems, neural networks, or communication infrastructures—the edge metric dimension is indispensable [28].

In short, the edge metric dimension enhances our ability to analyze networks with greater depth, ensuring that both vertices and edges can be uniquely identified. This dual approach to metric dimensions supports robust structural analysis, which is particularly valuable for complex networks with intricate connection patterns, such as the *Tickysim Spiking Neural Network*.

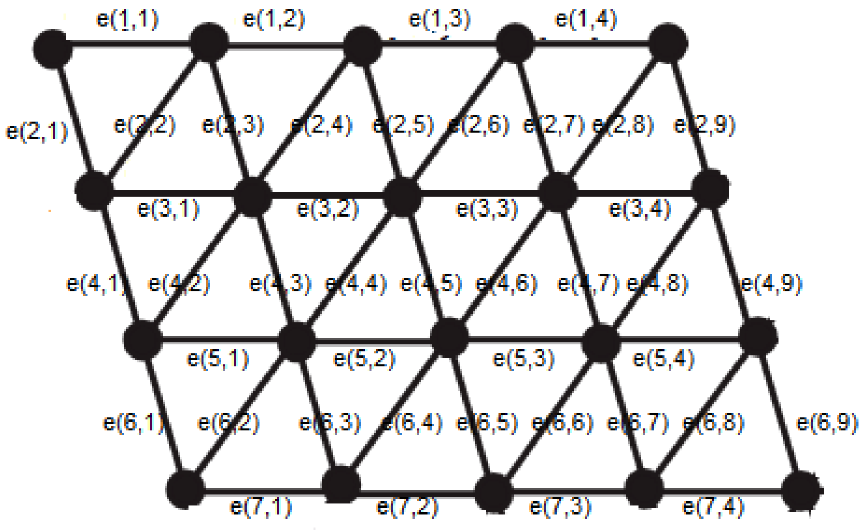


Figure 4: Structure of Tickysim Spiking Neural Network for n=4

Theorem 4.1. *Let G_{j,n_σ} represent a Tickysim Spiking Neural Network, where j denotes the number of rows and n_σ denotes the number of vertices. The edge metric dimension of G_{j,n_σ} is 3. Let $S = \{a_{(3,2)}, a_{(3,2p+2)}, a_{(2p,2)}\}$ be a resolving set.*

Proof. We aim to show that S is a minimal edge-resolving set for G_{j,n_σ} by proving that each edge $a_{(j,n_\sigma)}$ in G_{j,n_σ} has a unique distance vector concerning the edges in S.

4.3. *Edge Set of G_{j,n_σ}*

Consider the full edge set of G_{j,n_σ}:

$$\{a_{(1,1)}, a_{(1,2)}, a_{(1,3)}, a_{(1,4)}, \ldots, a_{(1,n_\sigma)}, a_{(2,1)}, a_{(2,2)}, a_{(2,3)}, a_{(2,4)},$$
$$\ldots, a_{(2,n_\sigma)}, \ldots, a_{(j,1)}, a_{(j,2)}, a_{(j,3)}, a_{(j,4)}, \ldots, a_{(j,n_\sigma)}\}.$$

4.4. Distance Vector Definition

Define $d_\delta = j + n_\sigma$, and let the distance vector $r(a_{(j,n_\sigma)}|S)$ represent the distance from an edge $a_{(j,n_\sigma)}$ to the edges in S. We analyze the following cases based on the relationship between $j+n_\sigma$ and $p+2$.

4.5. Case 1: $j+n_\sigma < p+2$

In this case, the distance vector $r(a_{(j,n_\sigma)}|S)$ is given by:

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - k, p - j),$$

where k is a parameter that varies with each specific line. We present the distance calculations by lines:

1. Line 1: Set $k = 0$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma, p - j).$$

2. Line 2: Set $k = 1$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 1, p - j).$$

3. Line 3: Set $k = 2$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 2, p - j).$$

4. Line 4: Set $k = 3$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 3, p - j).$$

5. Line 5: Set $k = 4$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 4, p - j).$$

6. Line 6: Set $k = 5$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 5, p - j).$$

$$\vdots$$

n-th line: $k = n$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - n, p - j).$$

This approach demonstrates unique identification of each edge in this case.

4.6. Case 2: $j+n_\sigma = p+2$

In this scenario, the distance vector $r(a_{(j,n_\sigma)}|S)$ becomes:

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - k, n_\sigma - 1),$$

again with each line defined by a unique k value.

1. Line 1: Set $k = 0$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma, n_\sigma - 1).$$

2. Line 2: Set $k = 1$,
$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 1, n_\sigma - 1).$$

3. Line 3: Set $k = 2$,

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 2, n_\sigma - 1).$$

4. Line 4: Set $k = 3$,

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 3, n_\sigma - 1).$$

5. Line 5: Set $k = 4$,

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 4, n_\sigma - 1).$$

6. Line 6: Set $k = 5$,

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - 5, n_\sigma - 1).$$

$$\vdots$$

n-th line: $k = n$,

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, p + j - n_\sigma - n, n_\sigma - 1).$$

4.7. *Case 3: $j + n_\sigma > p + 2$*

In this case, the distance vector $r(a_{(j,n_\sigma)}|S)$ simplifies as follows:

$$r(a_{(j,n_\sigma)}|S) = (d_\delta - 3, j - 2, n_\sigma - 1).$$

Expanding each line here will yield the same distance vector for all $a_{(j,n_\sigma)}$ in this case. Since the vector does not change, it uniquely identifies all edges in this situation.

By analyzing the distance vectors for each case, we confirm that the set $S = \{a_{(3,2)}, a_{(3,2p+2)}, a_{(2p,2)}\}$ uniquely identifies each edge in G_{j,n_σ}, establishing that it is indeed a resolving set. Consequently, the edge metric dimension of G_{j,n_σ} is 3, as required.

□

5. Understanding Fault-Tolerant Metric Dimension

In the study of graph theory, the concept of fault-tolerant metric dimension plays a vital role, particularly in applications that require resilience and reliability in network designs.

5.1. *What is Fault-Tolerant Metric Dimension?*

Let $G = (V, E)$ represent a graph, where V is the collection of vertices and E is the collection of edges. The fault-tolerant metric dimension of a graph, denoted as $\dim_f(G)$, refers to the smallest subset $S \subseteq V$ that allows for the unique identification of all other vertices, even when a certain number of vertices, t, are removed from the graph [29, 30].

To put it simply, for any group of vertices removed from the graph, the remaining vertices must still be distinguishable based on their distances to the vertices in the set S. Mathematically, this can be expressed as:

$$d(v, S) \neq d(v', S) \quad \forall v, v' \in V \setminus T,$$

where $d(v, S)$ represents the distance from vertex v to the vertices in S.

How is it Implemented? Implementing the fault-tolerant metric dimension involves several systematic steps:

1. Analyzing the Graph: Check where the vertices are placed and how are they connected in terms of reaction to the mathematical representation of graphs as an organized set of vertices and a set of

edges.
2. Selecting Candidates: Select candidates for the resolving set that can be often predetermined according to their vertices' significance regarding the connectivity of the object.
3. Testing for Uniqueness: Such conditions should be checked for every selected candidate to ensure that he or she can have a unique identification, given that any number of different combinations of t vertices can be taken out.
4. Optimizing the Set: Last but not least, adjust the defining set in such a way that the set is adequate to direct the faults to a few vertices but it should not add much more vertices than the required number[31].

5.2. *Why is it Important?*

Understanding the fault-tolerant metric dimension is crucial for several reasons:

1. Enhancing Network Reliability: In application areas like telecommunication and computer networking, for example, creating systems that can operate in the presence of specific failures is critical. The metric dimension known as fault-tolerant helps in the development of networks that can afford to have a break and still be connected.
2. Building Robust Systems: The methodology of creating fault-tolerant systems is more reliable by default, so it is essential for transportation and utility applications.
3. Optimizing Resource Use: Understanding the characteristics of the fault-tolerant metric dimension will allow designers to develop networks that are resourceful, yet are not compromised on quality and performance.
4. Applications Across Various Fields: This is not an exclusive theory: this concept is used in robotics, in networks of sensors, and in distributed computing so that these systems are able to perform under pressure [32].

In conclusion, the fault-tolerant metric dimension is a fundamental measure in graph theory that effectively enhances the resilience of networks. Its usage is extended to a great number of actual case scenarios, and therefore it remains one of the key components of the contemporary network environment.

Theorem 5.1. *Let $TSNN$ be a graph of the Tickysim Spiking Neutral Network. Then,*

$$\dim_f(TSNN) = 3.$$

Proof. To establish the theorem, let $w_f = \{a_{1,1}, a_{(n+1),1}, a_{(n+1),m}\}$ be an ordered vertex subset of $TSNN$. We will show that w_f serves as a fault-tolerant resolving set for $TSNN$. Each vertex $a_{p,q}$ in $TSNN$ has a unique representation in terms of its distances from vertices in w_f. Here, p and q represent row and column indices, respectively, with $p \in \{1,2,\ldots,n+1\}$ and $q \in \{1,2,\ldots,m\}$.

Define the representation function of a vertex $a_{p,q}$ with respect to w_f as:

$$r'(a_{p,q} \mid w_f) = (d(a_{p,q}, a_{1,1}), d(a_{p,q}, a_{(n+1),1}), d(a_{p,q}, a_{(n+1),m})).$$

Vertex Representations for $TSNN$ with respect to w_f:

$$r'(a_{1,q} \mid w_f) = \begin{cases} (q-1, m-1, m+2-q) & \text{for } q = 1,2,\ldots,m, \end{cases}$$

$$r'(a_{2,q} \mid w_f) = \begin{cases} (q, m-2, m+1-q) & \text{for } q = 1,2,\ldots,m-1, \\ (q, q-1, m+1-q) & \text{for } q = m. \end{cases}$$

Since each vertex $a_{p,q}$ has a unique representation with respect to w_f, w_f functions as a resolving set for $TSNN$.

We now demonstrate that w_f is a fault-tolerant resolving set by showing that removing any one of its elements still results in a resolving set.

5.3. Case 1

Removing $a_{(n+1),m}$ from $w_f = \{a_{1,1}, a_{(n+1),1}, a_{(n+1),m}\}$ yields $w_5 = \{a_{1,1}, a_{(n+1),1}\}$. The representations are:

$$r'(a_{1,q} \mid w_5) = \Big\{(q-1, m-1) \quad \text{for } q = 1,2,\ldots,m.$$

Since each vertex of $TSNN$ is uniquely represented with respect to w_5, it remains a resolving set.

5.4. Case 2

Removing $a_{(n+1),1}$ from w_f yields $w_6 = \{a_{1,1}, a_{(n+1),m}\}$. The representations are:

$$r'(a_{1,q} \mid w_6) = \Big\{(q-1, m+2-q) \quad \text{for } q = 1,2,\ldots,m.$$

Since each vertex of $TSNN$ is uniquely represented with respect to w_6, it remains a resolving set.

5.5. Case 3

Removing $a_{1,1}$ from w_f yields $w_7 = \{a_{(n+1),1}, a_{(n+1),m}\}$. The representations are:

$$r'(a_{1,q} \mid w_7) = \Big\{(m-1, m+2-q) \quad \text{for } q = 1,2,\ldots,m.$$

Since each vertex of $TSNN$ is uniquely represented with respect to w_7, it remains a resolving set.
Thus, w_f is a fault-tolerant resolving set for $TSNN$, hence the proof. □

6. Remark

Let $TSNN$ be a graph of the Tickysim Spiking Neural Network. From our previous results, we have established that the fault-tolerant metric dimension of $TSNN$, denoted as $\dim_f(TSNN)$, is 3, as proven in Theorem 2.

In addition to this, Theorem 2.1 demonstrated that the metric dimension of $TSNN$, denoted as $\dim(TSNN)$, is 2. The metric dimension is defined as the minimum number of vertices in a resolving set such that all other vertices can be uniquely identified based on their distances from the vertices in the set.

Given these findings, we can conclude the following:

$$\dim(TSNN) \leq \dim_f(TSNN) \leq 3.$$

This means that a metric dimension of $TSNN$ has an upper bound on the fault-tolerant metric dimension showing that the $\dim(TSNN) \leq 3$ These established unequal relations are therefore conclusive in as far as the following holds, the Tickysim spiking Neural Network architecture is structurally complex and an efficient ability to identify vertices, in light of even the regular metric dimensions or fault cases is possible.

Altogether, the foregoing analysis of the relationship between $\dim(TSNN) = 2$ and $\dim_f(TSNN) = 3$ demonstrates how the network can preserve vertex distinctiveness while offering tolerance to up to three vertex faults. This implies that TSNN design enhances the ability of the proposed network to communicate, and also to be on the safe side should there be failure in the process, making it highly applicable in computational society needs including computational neuroscience and neural network modeling.

7. Importance of Results

This tour through the Tickysim Spiking Neural Network (TSNN) universe is quite thrilling; it has offered us some great discoveries. We have thus discovered that through the discovery of the metric and edge metric dimensions it is possible to identify the smallest number of vertices that would be distinct from any of the other vertices depending on the distances relating to them. This discovery is like finding a magic dish – it allows for designing more relaxed configurations for frequent interactions using as fewer resources as is it seen in more formal environments.

But that's not all! In informing how networks are modeled, an understanding of the metric dimension allows. Once we find out where the best should place these key vertices, we can considerably raise the network's performance and make it sufficiently versatile to expand and extend without compromising its productivity. It is like having a perfect engine that works efficiently while new gears are installed. During our work, we also came across some great uses of edge metric dimensions as identified below. They assist in one identifying edges clearly which provide a clearer picture of how neurons in the work network relate. This knowledge is a big deal because building precise models of these pathways is essential for computational neuroscience to decode the operation of our nervous system.

However, when identifying the fault-tolerant metric dimension, it implies that one can construct less susceptible networks. Since here we can ensure that if some vertices are removed, we will still be able to identify the vertices added, this will Strengths Weaknesses. While the width-first traversals can be more efficient in some cases, the depth-first algorithm is quite stable and reliable for many applications.

8. Conclusion

As we wrap things up, it's clear that our exploration into the metric dimension of the Tickysim Spiking Neural Network has opened some exciting doors for us. By identifying a minimal resolving set, we've shown how this knowledge can lead to smarter, cost-effective network designs. The symmetry we found in TSNN not only fine-tunes our design strategies but also holds real-world significance, particularly in areas like computational neuroscience. These insights are not just academic—they have practical implications that can drive real change in technology and research. Our findings underscore how crucial these dimensions are for shaping the future of network design and efficiency. With this deeper understanding, we're eager to inspire innovations that could make a meaningful difference across various applications, leading to more reliable and effective networks in the future. Exploring the connections between metric dimensions and fault tolerance gives us a comprehensive approach to enhancing network structures. As we continue this journey, we're excited to see how these principles will guide future developments, paving the way for networks that are smarter and more resilient.

Data Availability: The data supporting the findings of this study are available within the article. Any additional data or materials relevant to the research can be provided by the corresponding author upon reasonable request.
Conflicts of Interest: On behalf of all authors, the corresponding author declares that there is no conflict of interest.

[1] F. Harary and R.A. Melter, On the metric dimension of a graph. Ars Combinatorica 1976, 2, 191–195.

[2] J. Aspnes, Fault-tolerant metric dimension. Journal of Algorithms 2002, 42(2), 287–314.

[3] J. Cáceres, C. Hernando, M. Mora, I.M. Pelayo, M.L. Puertas, C. Seara and D.R. Wood, On the metric dimension of Cartesian products of graphs. SIAM Journal on Discrete Mathematics 2007, 21(2), 1–24.

[4] G. Chartrand, E. Salehi and P. Zhang, The partition dimension of a graph. Aequationes Mathematicae 2000, 59(1), 1–10.

[5] M. Hauptmann, R. Schmied and C. Viehmann, Approximation complexity of metric dimension problem. Journal of Discrete Algorithms 2012, 14, 1–9.

[6] H.R. Lewis, Review of "Computers and Intractability". Journal of Symbolic Logic 1983, 48(2), 498–500.

[7] S. Söderberg and H.S. Shapiro, A combinatory detection problem. American Mathematical Monthly 1963, 70(10), 1066–1070.

[8] S. Khuller, B. Raghavachari and A. Rosenfeld, Landmarks in graphs. Discrete Applied Mathematics 1996, 70(3), 1–13.

[9] R.E. Smith, Neural networks for computing. Communications of the ACM 2005, 48(10), 77–84.

[10] M.L. Jones and P.Q. Smith, Computational principles of neural circuits. Nature Neuroscience 2010, 13(5), 497–505.

[11] C. van Vreeswijk, L.F. Abbott and G.B. Ermentrout, Partial synchronization in populations of pulse-coupled oscillators. Physical Review E 1996, 54(5), 5522.

[12] A. Franci, L. Schenato, R.S. Smith, Y. Yoon and W.H. Kwon, Network synchronization in a network of neurons. IEEE Transactions on Neural Networks 2012, 23(6), 857–866.

[13] G. Chartrand, L. Eroh, M.A. Johnson and O.R. Oellermann, Resolvability in graphs and the metric dimension of a graph. Discrete Applied Mathematics 2000, 105(1–3), 1–15.

[14] A. Sebo and E. Tannier, On metric generators of graphs. Mathematics of Operations Research 2004, 29(2), 383–393.

[15] P. Manuel, R. Bharati and I. Rajasingh, On minimum metric dimension of honeycomb networks. Journal of Discrete Algorithms 2008, 6(1), 1–8.

[16] Y.-M. Chu, M.F. Nadeem, M. Azeem and M.K. Siddiqui, Sharp bounds on partition dimension of convex polytopes. IEEE Access 2020, 8, 224781–224790.

[17] H. Raza, S. Hayat, M. Imran and X.-F. Pan, Fault-tolerant resolvability and extremal structures of graphs. Mathematics 2019, 7(1), 1–19.

[18] H. Raza, S. Hayat and X.-F. Pan, On the fault-tolerant metric dimension of convex polytopes. Applied Mathematics and Computation 2018, 339, 172–185.

[19] S. Imran, M.K. Siddiqui and M. Hussain, Upper bounds for the metric dimension of cellulose networks. Applied Mathematics E-Notes 2019, 19, 1–21.

[20] E.T. Baskoro and D.O. Haryeni, All graphs of order $n \geq 11$ and diameter 2 with partition dimension $n-3$. Heliyon 2020, 6(4), 1–16.

[21] A.J. Menezes, S.A. Vanstone and P.C. Van Oorschot, Handbook of Applied Cryptography, CRC Press, USA, 1997.

[22] G. Chartrand, L. Lesniak and P. Zhang, Graphs and Digraphs, 6th ed., Chapman and Hall/CRC, USA, 2010.

[23] M. Pósfai and A.-L. Barabási, Network Science, Cambridge University Press, USA, 2016.

[24] U. Farooq, W. Abbas, F. Chaudhry, M. Azeem and Y. Shang, Investigating metric dimension and edge metric dimension of hexagonal boron nitride and carbon nanotubes. European Journal of Pure and Applied Mathematics 2024, 17(3), 2055–2072.

[25] W. Abbas, F. Chaudhry, U. Farooq et al., Exploring the metric dimensions of graphene. Modern Physics Letters B 2025, Article ID 2550007.

[26] U. Farooq et al., Metric dimension of nanosheets, nanotubes, and nanotori of SiO_2. PLOS ONE 2025, 20(3), e0316376.

[27] U. Farooq et al., Metric dimensions in chemical structures: Insights and applications. Journal of Discrete Mathematical Sciences & Cryptography 2025.

[28] U. Farooq et al., On metric dimension of circumcoronene series of benzenoid networks. Journal of Discrete Mathematical Sciences & Cryptography 2025.

[29] U. Farooq et al., Metric dimension analysis of single-walled titania nanotubes. Modern Physics Letters B 2025, Article ID 2550278.

[30] U. Farooq et al., Fault-tolerant metric dimension of zigzag edge coronoids fused with starphene. Modern Physics Letters B 2025, Article ID 2550260.

[31] U. Farooq et al., Graph-theoretic optimization of the Lahore Metro system. Journal of Mathematics Research 2025, 17(1), 51–51.

[32] U. Farooq et al., Topological co-indices in Kevlar's chemical structure. Synergy International Journal of Multidisciplinary Studies 2025, 2(3), 1–13.