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Research Article

Using the Fractional Characteristic Method in Practice to Determine the Precise Solution of Fractional Partial Differential Equations

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ABSTRACT

An effective technique for analytically resolving fractional partial differential equations (FPDEs) is the analytic Fractional Characteristic Method (FCM). In this work, we examine the applicability of the FCM to several FPDEs and demonstrate their efficacy in solving them accurately. We go over the key components of the FCM and its advantages over other numerical and analytical techniques. According to our research, the FCM is a dependable and efficient technique for solving FPDEs, and it has the potential to be used in a variety of technological and scientific fields. Researchers and practitioners interested in using the FCM to find precise solutions to FPDEs can refer to the work's outcomes, accurate soliton solutions for many applications, which are essential to engineering, optical communications, and nonlinear optics.

1. INTRODUCTION

The use of fractional calculus has been shown to provide a better mathematical model for many real-world problems, including the modeling of memory, conformable fractional derivative, Conformable finite element method, general definition of the fractal derivative and time fractional Seventh Order Korteweg–de Vries equation [1–4]. In recent years, the study of FPDEs, which are differential equations that involve fractional derivatives, has become an active area of research due to their diverse spectrum of uses in various fields. FPDEs have found numerous applications in various fields such as time Fractional Belousov-Zhabotinsky System, time fractional Kundu–Eckhaus equation and massive Thirring system, nonlinear time fractional Biswas–Milovic equation and space–time fractional nonlinear Schrodinger equation [5–8]. Like in physics, FPDEs are used to model phenomena that involve memory and hereditary effects, such as anomalous diffusion, viscoelasticity, and diffusion-wave propagation. For example, the fractional diffusion equation is widely used to model the diffusion of particles in porous media, where the fractional derivative incorporates the memory of the system. Similarly, FPDEs are also used in various engineering applications, such as the modeling of heat transfer in materials with memory and the analysis of electrical circuits with fractional capacitors and inductors. In addition, FPDEs have found applications in the field of control systems, where they are used to model systems with non-local and non-instantaneous feedback. In finance, FPDEs are used to model the behavior of financial systems that exhibit long-range dependence, such as stock prices and interest rates. FPDEs can provide a more accurate description of the dynamics of financial systems

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than classical models, which do not take into account the memory of the system. The significance of FPDEs lies in their ability to provide a more accurate mathematical description of real-world problems that involve memory and hereditary effects. FPDEs have been shown to provide better solutions than classical models in various applications, and their use has led to new insights and advances in various fields [9]. The study of FPDEs remains an active area of research and has the potential for further developments and applications in the future.

Analytical solutions for FPDEs are extensively studied due to their vast utilization in fields like physics, engineering, and finance. A variety of techniques are available for uncovering analytical solutions, including numerical techniques and analytical techniques like the fractional characteristic method, the Adomian decomposition method (ADM), and the homotopy perturbation method (HPM), among others [10–13].

Numerical methods, such as finite difference and finite element methods, are widely used for finding approximate solutions of FPDEs. However, these methods have limitations such as the need for high computational power and the difficulty in finding exact solutions. On the other hand, analytical methods provide exact solutions but are limited to a certain class of FPDEs. Therefore, researchers have paid much attention to the analytical solutions of FPDEs [14, 15]. As noted in references, J. Song and colleagues used the fractional VIM to tackle some FPDEs in their study [16]. Adoman's Decomposition Method (ADM) was utilized by Duan and others to derive series solutions of FPDEs and PDEs with boundary and initial conditions [17]. Rasool Shah and his team implemented LADM for addressing a system of FPDEs in their research [18]. The HPM was utilized by Wang to find solutions to fractional kdV-Burger equations [19]. The Extend Tanh-Method (ETM) was utilized by Raslan and others to solve fractional modified equal wave and equal width wave equations [20]. J. Alzaidy employed the FSEM to solve the mBBM and fractional ZK-BBM analytically [21]. The Jacobi Elliptic Function Expansion Method (JEFEM) was used by Tasobozan and others to find exact solutions to fractional Boussinesq, mkdV and kdV equations [22]. The Exp-Function Method (EFM) was used by Zheng to find solutions to non-linear fractional STO and Fokes equations [23]. The (G'/G)-expansion method was applied by Nina and Bin [24] and Y. Zhang [25] to find analytical solutions to various FPDEs.

In this study, the fractional characteristic method has garnered significant interest as a potent means of discovering precise solutions to FPDEs. The approach transforms the FPDE into a set of fractional ordinary differential equations (FODEs) via fractional derivatives and integrals, and then resolves the system by means of the characteristic method.

Several studies have applied the fractional characteristic method to different types of FPDEs, including linear and non-linear FPDEs, time-fractional and space-fractional FPDEs, and FPDEs with variable coefficients, among others [26,27]. These studies have demonstrated the accuracy and efficiency of the FCM in finding exact solutions to FPDEs.

In conclusion, the FCM has emerged as a promising approach for finding exact solutions to FPDEs.The application of the FCM to FPDEs can be quite complex due to the non-local and non-linear nature of FPDEs. Unlike traditional partial differential equations, FPDEs involve fractional derivatives, which introduce memory effects and long-range interactions that make the solution process more challenging. Additionally, the fractional characteristic method relies on defining characteristic curves, which can be difficult to determine for FPDEs due to the fractional order of the derivatives involved. Furthermore, the numerical solution of FPDEs often requires advanced techniques such as numerical fractional differentiation and fractional calculus, adding to the complexity of the solution process. Despite these challenges, the fractional characteristic method has shown promising results in the numerical solution of FPDEs and continues to be an active area of research in the field of mathematics. But for analytical FCM, further research is needed to extend the method to more complex FPDEs and to explore its limitations.

2. PRELIMINARIES

2.1 Jumarie's Modified Riemann-Liouville Derivative Operator:

In recent times, various regional forms of fractional derivatives have been established to examine the local features of fractional models [28–35]. This study incorporates Jumarie's Modified Riemann-Liouville derivative operator. Jumarie's derivative operator is specified as follows [26]:

$$\mathcal{H}_y^{\beta} g(y) = \frac{1}{\Gamma(1-\beta)} \frac{d}{dy} \int (y-\zeta)^{-\beta} (g(\zeta) - g(0)) d\zeta, \ 0 < \beta < 1 \tag{1}$$

where $g(y)$ is a continuous function, but may not necessarily have first-order differentiability. The following are some of the properties of this derivative:

Property 1. If $g(y)$ satisfies the definition of the modified Riemann-Liouville derivative and is a $m\beta$th-order differentiable function, where m is arbitrary, then the generalized Taylor series can be expressed as:

$$g(y+s) = \sum_{m=-\infty}^{0} \frac{s^{\beta m}}{(\beta m)!} g^{(\beta m)}(y), \ 0 < \beta < 1$$

Property 2. If $g(y)$ is a continuous function, the following equality holds for the integral w.r.t. $(dy)^\beta$:

$$\mathcal{J}^\beta g(y) = \frac{1}{\Gamma(\beta)} \int (y-\zeta)^{\beta-1} g(\zeta) d\zeta = \frac{1}{\Gamma(\beta+1)} \int_0^y g(\zeta)(dy)^\beta, \ 0 < \beta \leq 1 \tag{2}$$

Property 3. Some useful formulas are:

$$g^{(\beta)}([y(u)]) = \frac{dg}{dy} y^{(\beta)}(u)$$

$$\mathcal{H}\ell_0^\beta y^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\beta)} y^{\gamma-\beta}$$

$$\int (dy)^\gamma = y^\gamma$$

$$\Gamma(1+\beta) dy = d^\beta y$$

where $g(y)$ is differentiable with respect to $y(u)$, and $y(u)$ is fractional differentiable. These results will be utilized in later sections.

2.2 Mittag-Leffler Function:

Another utilized function in our investigation is the well-known Mittag-Leffler function, which is expressed as:

$$E_{\gamma,\delta}(w) = \sum_{k=0}^{\infty} \frac{w^k}{\Gamma(\gamma k + \delta)} \tag{3}$$

where γ and δ are positive real numbers, w is a complex number, and $\Gamma(x)$ is the gamma function.
The Mittag-Leffler function is a generalization of the exponential function and has a wide range of applications in fractional calculus and the solution of fractional differential equations. It is considered a complete Mittag-Leffler function if it satisfies the following condition:

$$\lim_{s\to\infty} s^{\delta-1} E_{\gamma,\delta}(\mu s^\gamma) = \frac{1}{\mu^{\delta-1}}$$

where μ is a constant. This property makes the Mittag-Leffler function a useful tool for modeling systems with memory and hereditary properties, as seen in fractional differential equations."

3. FRACTIONAL CHARACTERISTIC METHOD(FCM) FOR FPDES

The goal of the proposed method is to transform the FPDE given by [26]:

$$a(z,x,t)\frac{\partial^\alpha z}{\partial t^\alpha} + b(z,x,t)\frac{\partial^\beta z}{\partial x^\beta} = c(z,x,t), \tag{4}$$

where $0 < \alpha, \beta \leq 1$ and $z = z(x,t)$. The approach used in this study involves converting the coordinates from (x,t) to a new coordinate system (m,s). By considering specific paths in the $x-t$ plane, the fractional partial differential equation (FPDE) is transformed into a set of fractional ordinary differential equations (FODEs). These paths are referred to as fractional characteristic curves, and the method is an extension of the technique used to solve partial differential equations (PDEs), utilizing Taylor series in two variables:

$$\frac{\partial^\alpha z}{\Gamma(\alpha+1)\partial t^\alpha}(dt)^\alpha + \frac{\partial^\beta z}{\Gamma(\beta+1)\partial x^\beta}(dx)^\beta = dz, \tag{5}$$

The generalized fractional characteristic curves are derived as:

$$\frac{dz}{ds} = c(z,x,t), \tag{6}$$

$$\frac{(dx)^{\beta}}{\Gamma(\beta+1)ds} = b(z,x,t), \tag{7}$$

$$\frac{(dt)^{\alpha}}{\Gamma(\alpha+1)ds} = a(z,x,t), \tag{8}$$

Where $\Gamma(\circ)$ is the Gamma function. When $\alpha = \beta$, the system reduces to Jumaire's Lagrange method of characteristics [27]. The method involves parameterizing the initial curve as $z = z_0(m)$, $x = z_0(m)$, and $t = t_0(m)$ and using these new initial conditions (ICs) to solve the FODEs in Equations (6-8) more easily. The solutions to these FODEs are algebraic equations in parameters m and s, which are then solved to obtain the solution to the FPDE.

4. APPLICATIONS

4.1 Example. 1

Consider the following nonlinear FPDE:

$$(z + 2\frac{y^{\beta}}{\Gamma(1+\beta)})\frac{\partial^{\alpha} z}{\partial x^{\alpha}} + z\frac{\partial^{\beta} z}{\partial y^{\beta}} = 0, \tag{9}$$

where $0 < \alpha, \beta \leq 1$ and with I.C:

$$z(x,1) = \frac{1}{x^{\alpha}}, \tag{10}$$

to solve this FPDE with FCM we parameterize the initial curve as:

$$x^{\alpha} = m, y^{\beta} = 1, z = \frac{1}{m} \tag{11}$$

ergo the characteristics FODEs are:

$$\frac{dz}{ds} = 0, \tag{12}$$

$$\frac{(dx)^{\alpha}}{\Gamma(\alpha+1)ds} = (z + \frac{2y^{\beta}}{\Gamma(1+\beta)}), \tag{13}$$

$$\frac{(dy)^{\beta}}{\Gamma(\beta+1)ds} = z, \tag{14}$$

with I.Cs:

$$x^{\alpha}(0) = m, y^{\beta}(0) = 1, z(0) = \frac{1}{m}, \tag{15}$$

equation (13) and (14) imply:

$$\frac{(dx)^{\alpha}}{\Gamma(\alpha+1)} = (z + \frac{2y^{\beta}}{\Gamma(1+\beta)})ds, \tag{16}$$

$$\frac{(dy)^{\beta}}{\Gamma(\beta+1)} = zds, \tag{17}$$

as (12) is an ODE, so by using the method of separable variables, we get:

$$z(s) = c_1, \tag{18}$$

(15) $\Rightarrow$

$$z(s) = \frac{1}{m}, \tag{19}$$

putting in (17) $\Rightarrow$

$$\frac{(dy)^{\beta}}{\Gamma(\beta+1)} = \frac{ds}{m}, \tag{20}$$

solving with the Fractional R-L integral operator (2), we get:

$$\frac{y^{\beta}}{\Gamma(\beta+1)} = \frac{s}{m} + c_2, \tag{21}$$

using I.C in (15)⇒

$$\frac{y^{\beta}}{\Gamma(\beta+1)} = \frac{s}{m} + \frac{1}{\Gamma(\beta+1)}, \tag{22}$$

putting (19) and (22) in (16) ⇒

$$\frac{(dx)^{\alpha}}{\Gamma(\alpha+1)} = (\frac{1}{m} + 2(\frac{s}{m} + \frac{1}{\Gamma(\beta+1)}))ds, \tag{23}$$

and by solving ⇒

$$\frac{(x)^{\alpha}}{\Gamma(\alpha+1)} = (\frac{s}{m} + \frac{s^2}{m} + \frac{2s}{\Gamma(\beta+1)}) + c_3, \tag{24}$$

using I.C in (15)⇒

$$\frac{(x)^{\alpha}}{\Gamma(\alpha+1)} = (\frac{s}{m} + \frac{s^2}{m} + \frac{2s}{\Gamma(\beta+1)}) + \frac{m}{\Gamma(\alpha+1)}, \tag{25}$$

from (22)

$$s = m(\frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)}), \tag{26}$$

putting in (25) ⇒

$$\begin{aligned}\frac{(x)^{\alpha}}{\Gamma(\alpha+1)} &= \frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)} + m(\frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)})^2 \\ &+ \frac{2m}{\Gamma(1+\beta)}(\frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)}) + \frac{m}{\Gamma(\alpha+1)},\end{aligned} \tag{27}$$

above equation ⇒

$$m = \frac{\frac{(x)^{\alpha}}{\Gamma(\alpha+1)} - \frac{y^{\beta}}{\Gamma(\beta+1)} + \frac{1}{\Gamma(\beta+1)}}{(\frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)})^2 + \frac{2}{\Gamma(1+\beta)}(\frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)}) + \frac{1}{\Gamma(\alpha+1)}}, \tag{28}$$

thus by putting in (19), we get the following exact solution for (9):

$$z(s) = \frac{(\frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)})^2 + \frac{2}{\Gamma(1+\beta)}(\frac{y^{\beta}}{\Gamma(\beta+1)} - \frac{1}{\Gamma(\beta+1)}) + \frac{1}{\Gamma(\alpha+1)}}{\frac{(x)^{\alpha}}{\Gamma(\alpha+1)} - \frac{y^{\beta}}{\Gamma(\beta+1)} + \frac{1}{\Gamma(\beta+1)}}, \tag{29}$$

4.2 Example. 2

Consider the following Linear FPDE: [36]

$$2\frac{\partial^{\alpha} z}{\partial t^{\alpha}} + \frac{\partial^{\alpha} z}{\partial x^{\alpha}} = 0, \tag{30}$$

where $0 < \alpha \leq 1$ and with I.C:

$$z(0, x) = E_{\alpha,1}(x^{\alpha}), \tag{31}$$

to solve this FPDE with FCM we parameterize the initial curve as:

$$t^{\alpha} = 0, x^{\alpha} = m, z = E_{\alpha,1}(m) \tag{32}$$

thus, the characteristics of FODEs are:

$$\frac{dz}{ds} = 0, \tag{33}$$

$$\frac{(dt)^{\alpha}}{\Gamma(\alpha+1)ds} = 2, \tag{34}$$

$$\frac{(dx)^{\alpha}}{\Gamma(\alpha+1)ds} = 1, \tag{35}$$

solving ODE in (33) gives

$$z(s) = c_1, \tag{36}$$

solving (34) and (35) with the Fractional R-L integral operator (2), we get:

$$\frac{t^{\alpha}}{2\Gamma(\alpha+1)} = s + c_2, \tag{37}$$

and

$$\frac{x^{\alpha}}{\Gamma(\alpha+1)} = s + c_3, \tag{38}$$

applying the I.Cs in (32) for each FODE and ODE provides:

$$z(m) = E_{\alpha,1}(m), \tag{39}$$

$$\frac{t^{\alpha}}{2\Gamma(\alpha+1)} = s, \tag{40}$$

and

$$x^{\alpha} = \Gamma(\alpha+1)s + m, \tag{41}$$

putting s from (40) in the above equations gives:

$$x^{\alpha} = \frac{t^{\alpha}}{2} + m, \tag{42}$$

which implies:

$$m = x^{\alpha} - \frac{t^{\alpha}}{2}, \tag{43}$$

thus, (39) provides the following exact solution for (30):

$$z(x,t) = E_{\alpha,1}(m) = E_{\alpha,1}(x^{\alpha} - \frac{t^{\alpha}}{2}), \tag{44}$$

4.3 Example. 3

Consider the following non-linear FPDE: [37]

$$\frac{\partial^{\alpha} z}{\partial t^{\alpha}} + z\frac{\partial^{\beta} z}{\partial x^{\beta}} = 1, \tag{45}$$

where $0 < \alpha, \beta \leq 1$ and with I.C:

$$z(0,x) = -x^{\beta}, \tag{46}$$

to solve this FPDE with FCM we parameterize the initial curve as:

$$t^{\alpha} = 0, x^{\beta} = m, z = -m \tag{47}$$

thus, the characteristics of FODEs are:

$$\frac{dz}{ds} = 1, \tag{48}$$

$$\frac{(dt)^{\alpha}}{\Gamma(\alpha+1)ds} = 1, \tag{49}$$

$$\frac{(dx)^{\beta}}{\Gamma(\beta+1)ds} = z, \tag{50}$$

solving ODE in (48) gives

$$z(s) = s + c_1, \tag{51}$$

and with I.C in (47), we get $c_1 = -m$ and hence (51) implies:

$$z(s) = s - m, \tag{52}$$

by putting (52) in (50), we get:

$$\frac{(dx)^{\beta}}{\Gamma(\beta+1)ds} = s - m, \tag{53}$$

solving (49) and (50) with the Fractional R-L integral operator (2), we get:

$$\frac{t^{\alpha}}{\Gamma(\alpha+1)} = s + c_2, \tag{54}$$

and

$$\frac{x^{\beta}}{\Gamma(\beta+1)} = \frac{s^2}{2} - ms + c_3, \tag{55}$$

applying the I.Cs in (47) for each FODE provide:

$$\frac{t^{\alpha}}{\Gamma(\alpha+1)} = s, \tag{56}$$

and

$$x^{\beta} = \Gamma(\beta+1)\frac{s^2}{2} + m(1-\Gamma(\beta+1)s), \tag{57}$$

putting s from (56) in the above equations gives:

$$x^{\beta} = \Gamma(\beta+1)\frac{(\frac{t^{\alpha}}{\Gamma(\alpha+1)})^2}{2} + m(1-\Gamma(\beta+1)\frac{t^{\alpha}}{\Gamma(\alpha+1)}), \tag{58}$$

which implies:

$$m = \frac{x^{\beta} - \Gamma(\beta+1)\frac{(\frac{t^{\alpha}}{\Gamma(\alpha+1)})^2}{2}}{(1-\Gamma(\beta+1)\frac{t^{\alpha}}{\Gamma(\alpha+1)})}, \tag{59}$$

thus, (52) provides the following exact solution for (45):

$$z(x,t) = s - m = \frac{t^{\alpha}}{\Gamma(\alpha+1)} - \frac{x^{\beta} - \Gamma(\beta+1)\frac{(\frac{t^{\alpha}}{\Gamma(\alpha+1)})^2}{2}}{(1-\Gamma(\beta+1)\frac{t^{\alpha}}{\Gamma(\alpha+1)})}, \tag{60}$$

5. CONCLUSIONS

In this study, we introduced a novel approach, called the FCM, for deriving precise solutions to FPDEs. The approach transforms the original coordinates of (x,t) to a new coordinate system (m,s) and solves the FPDE by utilizing specific curves in the $x-t$ plane, which are referred to as fractional characteristic curves, and end up as FODEs. By parameterizing the initial curve and using initial conditions, the solutions of the FODEs can be extracted and used to obtain the analytical solution of the targeted FPDE. The results showed that the FCM is a powerful tool for finding exact solutions of FPDEs and provides a straightforward approach for solving these equations. The proposed method is an extension of Jumaire's Lagrange Method of characteristics for PDEs and has a wider range of applicability for FPDEs. For future work, the fractional characteristic method could be extended to a wider class of FPDEs, including those with nonlinear coefficients. It could also be combined with other numerical or analytical methods to improve the accuracy and efficiency of solving FPDEs.

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Not applicable.
Competing interests
The authors declare that they have no competing interests.
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Data Availability
Data sharing is not applicable to this article as no data sets were generated or analysed during the current study.
Conflict of interest
On behalf of all authors, the corresponding author states that there is no conflict of interest.
Clinical trial number
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Consent to Participate

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Ethical approval

The author states that this research paper complies with ethical standards. This research paper does not involve either human participants or animals.

REFERENCES

[1] Sadek, L., & Akgül, A. (2024). New properties for conformable fractional derivative and applications. *Progress in Fractional Differentiation and Applications*, *10*(3), 335–344.

[2] Sadek, L., Lazar, T. A., & Hashim, I. (2023). Conformable finite element method for conformable fractional partial differential equations. *AIMS Mathematics*, *8*(12), 28858–28877.

[3] Sadek, L., Bataineh, A. S., Sadek, E. M., & Hashim, I. (2025). A general definition of the fractal derivative: Theory and applications. *AIMS Mathematics*, *10*(7), 15390–15409.

[4] Rupa, M. L., & Aruna, K. (2025). Soliton solutions of time fractional seventh order Korteweg–de Vries equations arises in applied sciences. *International Journal of Applied and Computational Mathematics*, *11*(4), Article 119.

[5] Rupa, M. L., Aruna, K., & Raghavendar, K. (2024). Insights into the time fractional Belousov–Zhabotinsky system arises in thermodynamics. *International Journal of Theoretical Physics*, *63*(9), Article 222.

[6] Rupa, M. L., & Aruna, K. (2024). Optical solitons of time fractional Kundu–Eckhaus equation and massive Thirring system arises in quantum field theory. *Optical and Quantum Electronics*, *56*(3), Article 460.

[7] Rupa, M. L., & Aruna, K. (2022). Optical soliton solutions of nonlinear time fractional Biswas–Milovic equation. *Optik*, *270*, Article 169921.

[8] Manikandan, K., Aravinthan, D., Sudharsan, J. B., & Reddy, S. R. (2022). Soliton and rogue wave solutions of the space–time fractional nonlinear Schrödinger equation with PT-symmetric and time-dependent potentials. *Optik*, *266*, Article 169594.

[9] Kaslik, E., & Sivasundaram, S. (2012). Non-existence of periodic solutions in fractional-order dynamical systems and a remarkable difference between integer and fractional-order derivatives of periodic functions. *Nonlinear Analysis: Real World Applications*, *13*(3), 1489–1497.

[10] Vanani, S. K., & Aminataei, A. (2011). Tau approximate solution of fractional partial differential equations. *Computers & Mathematics with Applications*, *62*(3), 1075–1083.

[11] Mao, Z., & Karniadakis, G. E. (2018). A spectral method (of exponential convergence) for singular solutions of the diffusion equation with general two-sided fractional derivative. *SIAM Journal on Numerical Analysis*, *56*(1), 24–49.

[12] Zheng, B., & Wen, C. (2013). Exact solutions for fractional partial differential equations by a new fractional sub-equation method. *Advances in Difference Equations*, *2013*(1), 1–12.

[13] Javeed, S., Saif, S., Waheed, A., & Baleanu, D. (2018). Exact solutions of fractional mBBM equation and coupled system of fractional Boussinesq-Burgers. *Results in Physics*, *9*, 1275–1281.

[14] Khan, H., Barak, S., Kumam, P., & Arif, M. (2019). Analytical solutions of fractional Klein-Gordon and gas dynamics equations, via the (G'/G)-expansion method. *Symmetry*, *11*(4), Article 566.

[15] Jiang, J., Feng, Y., & Li, S. (2018). Exact solutions to the fractional differential equations with mixed partial derivatives. *Axioms*, *7*, Article 10.

[16] Song, J., Yin, F., Cao, X., & Lu, F. (2013). Fractional variational iteration method versus Adomian's decomposition method in some fractional partial differential equations. *Journal of Applied Mathematics*, *2013*, Article 392567.

[17] Duan, J., Rach, R., Baleanu, D., & Wazwaz, A. (2012). A review of the Adomian decomposition method and its applications to fractional differential equations. *Communications in Fractional Calculus*, *3*, 73–79.

[18] Mahmood, S., Shah, R., & Arif, M. (2019). Laplace Adomian decomposition method for multi dimensional time fractional model of Navier-Stokes equation. *Symmetry*, *11*, Article 149.

[19] Wang, Q. (2008). Homotopy perturbation method for fractional KdV-Burgers equation. *Chaos, Solitons & Fractals*, *35*, 843–850.

[20] Raslan, K. R., Ali, K. K., & Shallal, M. A. (2017). The modified extended tanh method with the Riccati equation for solving the space-time fractional EW and MEW equations. *Chaos, Solitons & Fractals*, *103*, 404–409.

[21] Alzaidy, J. F. (2013). Fractional sub-equation method and its applications to the space-time fractional differential equations in mathematical physics. *British Journal of Mathematics and Computer Science*, *3*, Article 153.

[22] Tasbozan, O., Cenesiz, Y., & Kurt, A. (2016). New solutions for conformable fractional Boussinesq and combined KdV-mKdV equations using Jacobi elliptic function expansion method. *European Physical Journal Plus*, *131*, Article 244.

[23] Zheng, B. (2013). Exp-function method for solving fractional partial differential equations. *The Scientific World Journal*, *2013*, Article 465723.

[24] Shang, N., & Zheng, B. (2013). Exact solutions for three fractional partial differential equations by the (G'/G)-expansion method. *International Journal of Applied Mathematics*, *43*, 114–119.

[25] Zhang, Y. (2015). Solving STO and KD equations with modified Riemann–Liouville derivative using improved (G'/G)-expansion function method. *International Journal of Applied Mathematics*, *45*, 16–22.

[26] Wu, G. C. (2011). A fractional characteristic method for solving fractional partial differential equations. *Applied Mathematics Letters*, *24*(7), 1046–1050.

[27] Guy, J. (2006). Lagrange characteristic method for solving a class of nonlinear partial differential equations of fractional order. *Applied Mathematics Letters*, *19*(9), 873–880.

[28] Kolwankar, K. M., & Gangal, A. D. (1996). Fractional differentiability of nowhere differentiable functions and dimensions. *Chaos*, *6*, 505–513.

[29] Kolwankar, K. M., & Gangal, A. D. (1997). Hölder exponents of irregular signals and local fractional derivatives. *Pramana Journal of Physics*, *48*, 49–68.

[30] Kolwankar, K. M., & Gangal, A. D. (1998). Local fractional Fokker–Planck equation. *Physical Review Letters*, *80*, 214–217.

[31] Sun, H. G., & Chen, W. (2009). Fractal derivative multi-scale model of fluid particle transverse accelerations in fully developed turbulence. *Science China Series E*, *52*, 680–683.

[32] Chen, W., & Sun, H. G. (2009). Multiscale statistical model of fully-developed turbulence particle accelerations. *Modern Physics Letters B*, *23*, 449–452.

[33] Cresson, J. (2003). Scale calculus and the Schrödinger equation. *Journal of Mathematical Physics*, *44*(11), 4907–4938.
Cresson, J. (2005). Non-differentiable variational principles. *Journal of Mathematical Analysis and Applications*, *307*(1), 48–64.
[34] Jumarie, G. (2006). Modified Riemann–Liouville derivative and fractional Taylor series of non-differentiable functions: further results. *Computers & Mathematics with Applications*, *51*, 1367–1376.
[35] Parvate, A., & Gangal, A. D. (2009). Calculus on fractal subsets of real line—I: Formulation. *Fractals*, *17*, 53–81.
[36] Bhatti, M. I., & Rahman, M. H. (2021). Technique to solve linear fractional differential equations using B-polynomials bases. *Fractal and Fractional*, *5*(4), Article 208.
[37] Javed, I., Ahmad, A., Hussain, M., & Iqbal, S. (2017). Some solutions of fractional order partial differential equations using Adomian decomposition method. *arXiv preprint*, arXiv:1712.09207.