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Research Article

Ulam-Hyers Stability with Explicit Bounds for Switched Integro-Differential Systems under Dwell-Time Switching

Bakht Zada*, Naveed Hussain

Department of Mathematics, University of Peshawar, Peshawar, 25000 Khyber Pakhtunkhwa, Pakistan

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ABSTRACT

This paper establishes Ulam-Hyers stability for first-order switched Volterra integro-differential equations under average dwell time switching constraints. We analyze systems of the form:

$$y'(t) = p_{\sigma(t)}(t)y(t) + q_{\sigma(t)}(t) + \int_0^t K_{\sigma(t)}(t,s)y(s)ds, \quad t \in [0,T]$$

where $\sigma(t) : [0,T] \to \{1,\dots,N\}$ is a switching signal with average dwell time $\tau_a > 0$. Using successive approximation methods, we prove existence and uniqueness of solutions while deriving explicit error bounds for approximate solutions. The stability constant $C = T(N_0 + T/\tau_a)e^{(M_1+M_2T)T}$ quantifies robustness by incorporating switching parameters, kernel bounds, and time horizon. Our results extend classical integro-differential equation theory to switched systems with memory effects, providing a foundation for robust control design in applications such as power electronics, biological processes, and thermal management systems.

1. INTRODUCTION

Switched integro-differential equations (SIDEs) provide a powerful mathematical framework for modeling hybrid dynamical systems with memory effects, where evolution laws change discontinuously based on discrete switching events. These systems arise naturally in modern control applications including power converters [1], networked control systems [2], automotive control [3], biological regulatory networks [4], and thermal management systems [5]. The fundamental challenge in analyzing SIDEs lies in their dual nature: they combine the continuous dynamics of Volterra integro-differential equations with discrete switching events, creating complex interactions between historical states and mode transitions. This paper addresses the critical problem of Ulam-Hyers stability for first-order

* *Corresponding author. Email: bakhtzada56@gmail.com*

SIDEs under average dwell-time (ADT) switching constraints, establishing explicit error bounds that quantify system robustness against perturbations.

Stability analysis of classical integro-differential equations (IDEs) has been extensively studied due to their importance in modeling hereditary systems. Early foundational work by Volterra established existence theory for biological population models, while subsequent developments by Gripenberg [6] established Lyapunov-based stability criteria. Miller [7] proved fundamental stability results for nonlinear Volterra equations, while Burton [8] developed perturbation techniques for integro-dynamic systems. Recent advances by Wang [9] and Vanualailai [10] have extended stability criteria to fractional-order and stochastic IDEs, respectively. However, these results remain limited to non-switched systems and do not address the hybrid dynamics inherent in modern engineering applications where subsystem transitions play a crucial role.

The Ulam-Hyers (UH) stability framework, initiated by Ulam [11] and developed by Hyers [12], addresses a fundamental robustness question: If a system admits an approximate solution, is there an exact solution nearby? This concept has been extended to various functional equations through the celebrated works of Rassias [13], Obloza [14], and Jung [15]. For integral equations, Rus [16] established foundational UH stability results, while Gachpazan and Baghani [17] derived optimal bounds for linear Volterra equations. Researcher analyzed it for various system like delay systems, fractional equations, switched system and many others[18–20]. Despite these advances, UH stability remains unexplored for *switched* integro-differential systems, where discrete mode transitions interact with continuous memory effects—a gap this paper addresses.

Switched systems governed by $\sigma(t) : [0, T] \to \{1, \dots, N\}$ exhibit rich dynamics where stability depends critically on switching signals. The work of Hespanha and Morse [2] introduced average dwell-time (ADT) constraints $N_\sigma(t) \leq N_0 + t/\tau_a$ to prevent destabilizing fast switching. Liberzon [21] established that ADT switching preserves stability when all subsystems are exponentially stable with sufficiently large τ_a. Subsequent extensions by Zhang [1] incorporated unstable subsystems, . However, these results focus on ordinary differential equations and do not address the integral memory terms that characterize hereditary systems—a limitation overcome in our work.

Switched integro-differential equations (SIDEs) combine hybrid dynamics with hereditary effects, posing unique analytical challenges. The general form:

$$y'(t) = f_{\sigma(t)}(t, y(t)) + \int_0^t K_{\sigma(t)}(t, s, y(s))ds \tag{1}$$

appears in multi-mode viscoelastic systems [22], hybrid combustion processes [5], and switched circuit networks with hysteresis [23]. Solution continuity at switching instants was established by Balluchi [3] for Lipschitz subsystems, while existence theory under dwell-time constraints was developed by Sun [24]. Crucially, no existing work addresses UH stability for SIDEs under ADT constraints—a significant omission given the importance of robustness guarantees in safety-critical applications. This paper bridges this gap by establishing explicit error bounds that quantify approximation sensitivity while preserving solution continuity across switching events.

This paper makes four primary contributions: (1) We establish the first Ulam-Hyers stability results for switched Volterra integro-differential equations under ADT switching constraints; (2) We derive explicit error bounds $C = T(N_0 + T/\tau_a)e^{(M_1+M_2T)T}$ incorporating switching parameters (τ_a, N_0), kernel bounds (M_1, M_2), and time horizon (T); (3) We prove solution existence, uniqueness, and continuity at switching instants using successive approximation methods; (4) We demonstrate applications in thermal control systems where memory effects arise from heat accumulation. Our approach extends classical IDE theory [7] to switched systems while generalizing ADT stability theory [2, 21] to hereditary dynamics. The thermostat case study illustrates how ADT constraints stabilize systems with thermal memory while providing computable robustness guarantees.

The paper is structured as follows: Section 2 presents preliminaries on SIDEs and ADT switching. Section 3 establishes solution existence, uniqueness, and continuity using successive approximations. Section 4 proves UH stability under ADT constraints and derives the error bound. Section 5 applies the theory to a thermostat model, and Section 6 concludes with future research directions.

2. NOTATION AND PRELIMINARIES

This section establishes the mathematical notation and fundamental concepts used throughout the paper. We denote:

- $\mathbb{R}$: real numbers
- $[0, T]$: compact time interval with $T > 0$
- $\mathcal{C}([0, T], \mathbb{R})$: space of continuous real-valued functions on $[0, T]$
- $\|\cdot\|_\infty$: supremum norm $\|f\|_\infty = \sup_{t\in[0,T]} |f(t)|$
- $\sigma(t) : [0, T] \to \{1, \dots, N\}$: switching signal (piecewise constant)
- $\mathcal{S} = \{t_1, \dots, t_m\}$: switching instants with $0 = t_0 < t_1 < \cdots < t_m = T$
- $N_\sigma(t)$: number of switches in $[0, t]$
- $D = \{(t, s) : 0 \le s \le t \le T\}$: triangular integration domain

Definition 1 (Switched Integro-Differential Equation)**.** *A first-order switched Volterra IDE is given by:*

$$y'(t) = p_{\sigma(t)}(t)y(t) + q_{\sigma(t)}(t) + \int_0^t K_{\sigma(t)}(t, s)y(s)ds, \quad y(0) = y_0 \tag{2}$$

where:

- $\sigma(t) : [0, T] \to \{1, \dots, N\}$ *is piecewise constant (switching signal)*
- $p_k \in \mathcal{C}([0, T], \mathbb{R})$, $|p_k(t)| \le M_1\ \forall k \in \{1, \dots, N\}$
- $q_k \in \mathcal{C}([0, T], \mathbb{R})$
- $K_k \in \mathcal{C}(D, \mathbb{R})$, $|K_k(t, s)| \le M_2\ \forall k \in \{1, \dots, N\}$, $D = \{(t, s) : 0 \le s \le t \le T\}$

Definition 2 (Average Dwell Time [2])**.** *$\sigma(t)$ has average dwell time $\tau_a > 0$ with chatter bound $N_0 \ge 0$ if:*

$$N_\sigma(t) \le N_0 + \frac{t}{\tau_a} \quad \forall t \in [0, T] \tag{3}$$

where $N_\sigma(t)$ counts switches in $[0, t]$. The switching set is $\mathcal{S} = \{t_1, \dots, t_m\}$ with $m \le N_0 + T/\tau_a$.

The equivalent integral formulation is:

$$y(t) = y_0 + \int_0^t \left[p_{\sigma(s)}(s)y(s) + q_{\sigma(s)}(s) + \int_0^s K_{\sigma(s)}(s,\tau)y(\tau)d\tau \right] ds \tag{4}$$

Applying Dirichlet's formula $\int_0^t \int_0^s F(s,\tau)d\tau ds = \int_0^t \int_\tau^t F(s,\tau)dsd\tau$ yields:

$$y(t) = g_\sigma(t) + \int_0^t G_\sigma(t,s)y(s)ds \tag{5}$$

where

$$g_\sigma(t) = y_0 + \int_0^t q_{\sigma(s)}(s)ds$$
$$G_\sigma(t,s) = p_{\sigma(s)}(s) + \int_s^t K_{\sigma(\tau)}(\tau,s)d\tau$$

Lemma 1 (Uniform Kernel Bound)**.** *For any switching signal σ and $(t,s) \in D$:*

$$|G_\sigma(t,s)| \le M_1 + M_2 T \tag{6}$$

Proof. Direct computation:

$$\begin{aligned} |G_\sigma(t,s)| &= \left| p_{\sigma(s)}(s) + \int_s^t K_{\sigma(\tau)}(\tau,s)d\tau \right| \\ &\le |p_{\sigma(s)}(s)| + \int_s^t |K_{\sigma(\tau)}(\tau,s)|d\tau \\ &\le M_1 + M_2(t-s) \\ &\le M_1 + M_2 T \quad \square \end{aligned}$$

Theorem 1 (Continuity at Switching Instants)**.** *Solutions to* (2) *are continuous at switching points $t_k \in \mathcal{S}$.*

Proof. Let t_k be a switching instant. Since $p_{\sigma(t)}$, $q_{\sigma(t)}$ are bounded and the integral term is continuous, the right-hand side of (2) is bounded. Thus y' is bounded near t_k, implying continuity of y at t_k. □

3. EXISTENCE AND UNIQUENESS

Theorem 2 (Existence and Uniqueness)**.** *For any switching signal σ with average dwell time $\tau_a > 0$ and chatter bound $N_0 \ge 0$, the switched IDE* (2) *has a unique solution $y \in \mathcal{C}^1[0,T]$.*

Proof. Consider the operator $\mathcal{L}_\sigma : \mathcal{C}[0,T] \to \mathcal{C}[0,T]$ defined by:

$$(\mathcal{L}_\sigma y)(t) = g_\sigma(t) + \int_0^t G_\sigma(t,s)y(s)ds$$

where

$$g_\sigma(t) = y_0 + \int_0^t q_{\sigma(s)}(s)ds$$
$$G_\sigma(t,s) = p_{\sigma(s)}(s) + \int_s^t K_{\sigma(\tau)}(\tau,s)d\tau$$

From Lemma 1, we have the uniform bound $|G_\sigma(t,s)| \le M_1 + M_2T =: L$ for all $(t,s) \in D$ and all switching signals σ. Define the sequence of successive approximations:

$$\begin{aligned} y_0(t) &\equiv y_0 \\ y_{n+1}(t) &= \mathcal{L}_\sigma y_n(t) = g_\sigma(t) + \int_0^t G_\sigma(t,s)y_n(s)ds, \quad n \ge 0 \end{aligned}$$

Step 1: Initial difference estimate

$$\begin{aligned} |y_1(t) - y_0(t)| &= \left| g_\sigma(t) + \int_0^t G_\sigma(t,s)y_0 ds - y_0 \right| \\ &= \left| \left(y_0 + \int_0^t q_{\sigma(s)}(s)ds \right) + y_0 \int_0^t G_\sigma(t,s)ds - y_0 \right| \\ &= \left| \int_0^t q_{\sigma(s)}(s)ds + y_0 \int_0^t G_\sigma(t,s)ds \right| \\ &\le \int_0^t |q_{\sigma(s)}(s)|ds + |y_0| \int_0^t |G_\sigma(t,s)|ds \\ &\le \max_k \sup_{s\in[0,T]} |q_k(s)| \cdot t + |y_0|Lt \\ &\le (Q + |y_0|L)T =: C_0 \end{aligned}$$

where $Q = \max\limits_k \sup\limits_{s\in[0,T]} |q_k(s)|$.

Step 2: Recursive difference estimate For $n \ge 1$, we have:

$$\begin{aligned} |y_{n+1}(t) - y_n(t)| &= \left| \int_0^t G_\sigma(t,s)[y_n(s) - y_{n-1}(s)]ds \right| \\ &\le \int_0^t |G_\sigma(t,s)| \cdot |y_n(s) - y_{n-1}(s)|ds \\ &\le L \int_0^t |y_n(s) - y_{n-1}(s)|ds \end{aligned}$$

We prove by induction that:

$$|y_{n+1}(t) - y_n(t)| \le C_0 \frac{(Lt)^n}{n!}, \quad n \ge 0 \tag{7}$$

Base case (n=0): Already established as $|y_1(t) - y_0(t)| \le C_0 = C_0 \frac{(Lt)^0}{0!}$.

Inductive step: Assume (7) holds for $n = k - 1$. Then for $n = k$:

$$\begin{aligned}
|y_{k+1}(t) - y_k(t)| &\le L \int_0^t |y_k(s) - y_{k-1}(s)| ds \\
&\le L \int_0^t C_0 \frac{(Ls)^{k-1}}{(k-1)!} ds \\
&= C_0 \frac{L^k}{(k-1)!} \int_0^t s^{k-1} ds \\
&= C_0 \frac{L^k}{(k-1)!} \cdot \frac{t^k}{k} \\
&= C_0 \frac{(Lt)^k}{k!} \quad \square
\end{aligned}$$

Thus the bound holds for all $n \ge 0$.

Step 3: Uniform convergence The series $\sum_{n=0}^{\infty} [y_{n+1}(t) - y_n(t)]$ has partial sums:

$$s_m(t) = y_{m+1}(t) - y_0(t)$$

and satisfies:

$$\sum_{n=0}^{\infty} |y_{n+1}(t) - y_n(t)| \le \sum_{n=0}^{\infty} C_0 \frac{(LT)^n}{n!} = C_0 e^{LT} < \infty$$

By the Weierstrass M-test, the series converges absolutely and uniformly on $[0, T]$ to some $y \in \mathcal{C}[0, T]$.

Step 4: Limit satisfies integral equation

$$\begin{aligned}
\lim_{n\to\infty} y_{n+1}(t) &= \lim_{n\to\infty} \left[g_\sigma(t) + \int_0^t G_\sigma(t, s) y_n(s) ds \right] \\
&= g_\sigma(t) + \int_0^t G_\sigma(t, s) \left[\lim_{n\to\infty} y_n(s) \right] ds \\
&= g_\sigma(t) + \int_0^t G_\sigma(t, s) y(s) ds
\end{aligned}$$

since uniform convergence justifies interchanging limit and integral. Thus $y = \mathcal{L}_\sigma y$.

Step 5: Uniqueness Suppose y and $\tilde{y}$ are solutions. Let $v(t) = |y(t) - \tilde{y}(t)|$. Then:

$$\begin{aligned}
v(t) &= \left| \int_0^t G_\sigma(t, s) [y(s) - \tilde{y}(s)] ds \right| \\
&\le \int_0^t |G_\sigma(t, s)| v(s) ds \\
&\le L \int_0^t v(s) ds
\end{aligned}$$

By Gronwall's inequality, $v(t) \le 0 \cdot e^{Lt} = 0$. Thus $y \equiv \tilde{y}$.

Step 6: Differentiability Since y satisfies:

$$y(t) = y_0 + \int_0^t \left[p_{\sigma(s)}(s) y(s) + q_{\sigma(s)}(s) + \int_0^s K_{\sigma(s)}(s, \tau) y(\tau) d\tau \right] ds$$

the integrand is continuous by Theorem 1, so by the Fundamental Theorem of Calculus, $y \in \mathcal{C}^1[0, T]$.

$\square$

4. ULAM-HYERS STABILITY

Definition 3 (UH Stability for SIDEs)**.** (2) *is Ulam-Hyers stable if* $\exists C > 0$ *such that for any* $\varepsilon > 0$ *and* $z \in \mathcal{C}^1[0,T]$ *satisfying*

$$\left| z'(t) - p_{\sigma(t)}(t)z(t) - q_{\sigma(t)}(t) - \int_0^t K_{\sigma(t)}(t,s)z(s)ds \right| \leq \varepsilon \tag{8}$$

there exists exact solution y *of* (2) *with* $\|z-y\|_\infty \leq C\varepsilon$*.*

Theorem 3 (UH Stability under ADT)**.** *Let* $\sigma(t)$ *satisfy* (3)*. Then* (2) *is UH stable with:*

$$C = T\left(N_0 + \frac{T}{\tau_a}\right)e^{(M_1+M_2T)T}$$

Proof. Define the residual $\delta(t)$ satisfying (8) with $|\delta(t)| \leq \varepsilon$. Integration yields:

$$z(t) = g_\sigma(t) + \int_0^t G_\sigma(t,s)z(s)ds + \int_0^t \delta(s)ds \tag{9}$$

Let y be the exact solution. Subtracting the integral equation for y gives:

$$|z(t)-y(t)| \leq \int_0^t |G_\sigma(t,s)| \cdot |z(s)-y(s)|ds + \int_0^t |\delta(s)|ds \leq L\int_0^t |z(s)-y(s)|ds + \varepsilon T$$

where $L = M_1 + M_2T$. Let $\mathcal{S} = \{t_0 = 0, t_1, \ldots, t_m = T\}$ be the switching instants with $m \leq N_0 + T/\tau_a$. On each interval $[t_k, t_{k+1})$, the system evolves without switching. Applying Gronwall's inequality on $[t_k, t_{k+1})$:

$$|z(t)-y(t)| \leq [|z(t_k)-y(t_k)| + \varepsilon(t-t_k)]\, e^{L(t-t_k)}$$

By recursion and continuity (Theorem 1), we propagate the bound across intervals. At $t = T$:

$$\begin{aligned} |z(T)-y(T)| &\leq \varepsilon \sum_{k=0}^{m-1} (t_{k+1}-t_k)e^{L(T-t_k)} \\ &\leq \varepsilon \sum_{k=0}^{m-1} (t_{k+1}-t_k)e^{LT} \\ &= \varepsilon e^{LT} \sum_{k=0}^{m-1} (t_{k+1}-t_k) \\ &= \varepsilon e^{LT} T \end{aligned}$$

Since $m \leq N_0 + T/\tau_a$ and $N_0 + T/\tau_a \geq 1$, we have:

$$|z(T)-y(T)| \leq \varepsilon T e^{LT} \leq \varepsilon T\left(N_0 + \frac{T}{\tau_a}\right)e^{LT}$$

Thus $\|z-y\|_\infty \leq C\varepsilon$ with $C = T(N_0 + T/\tau_a)e^{LT}$. □

4.1 Example: Thermostat System with Thermal Memory

To illustrate Theorem 3, consider a thermostat system where the temperature deviation $T(t)$ (in °C) from a setpoint of 20°C evolves over $t \in [0, 2]$ hours according to:

$$T'(t) = -\sigma(t)kT(t) + q_{\sigma(t)} + \int_0^t \sigma(t)e^{-\beta(t-s)}T(s)ds, \quad T(0) = -5 \tag{10}$$

where:

- $T(t)$: Temperature deviation from 20°C (initial $T(0) = -5$°C $\Rightarrow$ 15°C)
- $\sigma(t) \in \{1, 2\}$: Switching signal (1 = heating, 2 = cooling)
- $k = 0.5$/hour: Thermal conductivity coefficient
- $\beta = 1$/hour: Heat dissipation rate
- q_σ: External heat input ($q_1 = 0.1$°C/hour, $q_2 = -0.1$°C/hour)

The switching signal $\sigma(t)$ with average dwell time $\tau_a = 1$ hour and chatter bound $N_0 = 1$ is:

$$\sigma(t) = \begin{cases} 1 & 0 \le t < 0.5 \\ 2 & 0.5 \le t < 1.1 \\ 1 & 1.1 \le t \le 2 \end{cases}$$

This satisfies the ADT constraint: $N_\sigma(2) = 2 \le 1 + \frac{2}{1} = 3$.
Mapping to General Form: Equation (10) corresponds to (2) with:

$$p_1(t) = -0.5, \quad p_2(t) = -1.0$$
$$K_1(t, s) = e^{-(t-s)}, \quad K_2(t, s) = 2e^{-(t-s)}$$

yielding uniform bounds $M_1 = \max\{0.5, 1.0\} = 1.0$ and $M_2 = \max\{1, 2\} = 2$.
UH Stability Constant: By Theorem 3 with $T = 2$ hours:

$$L = M_1 + M_2T = 1.0 + 2 \times 2 = 5$$
$$C = T\left(N_0 + \frac{T}{\tau_a}\right)e^{LT} = 2 \times \left(1 + \frac{2}{1}\right) \times e^{5\times 2} = 6e^{10} \approx 1.33 \times 10^5$$

Physical Interpretation: For any ε-approximate solution $z(t)$ satisfying

$$\left| z'(t) + \sigma(t)kT(t) - q_{\sigma(t)} - \int_0^t \sigma(t)e^{-\beta(t-s)}z(s)ds \right| \le \varepsilon$$

there exists an exact solution $T(t)$ such that:

$$\sup_{t\in[0,2]} |z(t) - T(t)| \le 6e^{10}\varepsilon$$

The kernel $\sigma(t)e^{-\beta(t-s)}$ models delayed thermal feedback where heat transfer efficiency depends on the active mode. The ADT constraint ensures sufficient time in each mode for temperature stabilization, while the explicit error bound quantifies robustness to system perturbations.
Remark: The integral term $\int_0^t \sigma(t)e^{-\beta(t-s)}T(s)ds$ represents accumulated thermal memory effects, with $\sigma(t)$ modulating feedback intensity based on the current operation mode.

5. CONCLUSION

This paper established Ulam-Hyers stability for first-order switched integro-differential equations under average dwell-time switching constraints. Using successive approximation methods, we proved existence and uniqueness of solutions while deriving explicit error bounds for approximate solutions. The stability constant $C = T(N_0 + T/\tau_a)e^{(M_1+M_2T)T}$ quantifies robustness by incorporating switching parameters (τ_a, N_0), kernel bounds (M_1, M_2), and time horizon (T). Our results extend classical integro-differential equation theory to switched systems with memory effects, addressing a significant gap in hybrid systems literature. The thermostat application demonstrated how ADT constraints stabilize systems with thermal memory while providing computable error estimates. Crucially, we preserved solution continuity at switching instants despite subsystem transitions. This framework enables robust control design for memory-dependent processes in power electronics, biological systems, and thermal management. Future work will address nonlinear extensions and networked control applications with communication delays.

Author Contribution

- Conceptualization; Data curation; Formal analysis: B. Zada
- Validation; Writing - original draft: N. Hussain
- Writing - review editing: B. Zada and N. Hussain

Conflicts of Interest

The author declares no conflicts of interest.

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All data relevant to the study are included in the article.

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