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Research Article

Double Mohand Integral Transform for Solving Wave Equations

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ABSTRACT

Many fields of the physical sciences like physics, chemistry and mathematics use partial differential equations to model a physical system. As a result, there are diverse techniques in the literature for the solution of partial differential equations which tries to attain symmetry. This paper proposes a fresh double transform method recognized as the double Mohand Transform Method. Besides the explanation and major remarks of the proposed double transform method, new results on partial derivatives are also given. Moreover, to validate the importance of the double Mohand Transform Method in solving systems of partial differential equations, a variety of symmetric examples on the wave equation with various properties are solved using this approach.

1. INTRODUCTION

Integral transforms are regarded the extreme convenient technique for divulging the magic confines of partial differential equations (PDEs). PDEs are the center of attention as one can explore multifarious captivating phenomena from the realms of mathematics, physics and several scientific fields with its aid [1–13]. These transformations can be used to alter the form of these equations and compute definitive solutions for the PDEs. Because of their effectiveness and simplicity, transform methods have been extensively studied and improved by many researchers.

Numerous integral transforms have been put into action for the purpose of solving both integral and PDEs. These single integral transformations, reduce partial differential equations into ordinary ones, while double integral transformations approaches reduce them into algebraic forms. With such transforms, exact solutions to the governing equations can be obtained without going through the processes of linearization or discretization [14-24].

It is believed that double transformations are more efficient than other numerical techniques for managing PDEs because they have consistently been applied to solve PDEs involving functions with multiple variables [25-28]. A number of widely recognized alterations of the double integral transform are also covered in the associated work, such as the Laplace integral transform, Shehu integral transform [29], Kamal integral transform [30], Sumudu integral transform [31–36], Elzaki integral transform [37],

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Laplace–Sumudu integral transform [38], and the ARA–Sumudu integral transform [39,40]. It is conceivable to consider each of these double integral transforms as a particular instance of the general double integral transform that Meddahi and colleagues [41] suggested. The double Mohand transform [DMT], a novel integral transform, was put forward by Ullah and Aggarwal [42]. They have successfully used DMT in determining exact solutions of differential equations.
This study aims to establish the application of the double Mohand transform technique in solving wave equations. The fundamental properties and theorems of DMT are reviewed and some DMT values for certain functions are obtained. Theorems along with new relations that involve partial derivatives of the functions were used to solve the PDEs. The results of current investigation are original since it applies DMT to wave equations for the first time.
In this article, we address the nonhomogeneous linear wave equation, expressed as

$$f_{tt}(x,t) = \alpha f_{xx}(x,t) + \beta f(x,t) + \omega(x,t), \tag{1}$$

along with the proper initial conditions

$$f(x,0) = f_1(x), \qquad f_t(x,0) = f_2(x), \tag{2}$$

and the appropriate boundary conditions

$$f(0,t) = g_1(t), \qquad f_x(0,t) = g_2(t), \tag{3}$$

and $f(0,0) = \varphi$, where $\omega(x,t)$ is the source term, $f(x,t)$ is an unknown function, and α, β and φ are constants. To show how effective this novel approach is, a simple formula to its corresponding equation was designed, and a number of applications were solved using this implementation.
This work is structured as follows: in Section 2, we introduce the elementary concepts and discuss the key characteristics of the DMT. Employing DMT to solve the wave equation and derive its exact solution is discussed in section 3. In section 4, a number of cases are solved and demonstrated using the DMT. The discussion is concluded in section 5.

2. DOUBLE MOHAND TRANSFORM (DMT)

2.1 Definition

For all values of x and t greater or equal to 0, the DMT of the function f is defined as [42]

$$M_2[f(x,t): u, v] = R(u,v) = u^2 v^2 \iint_0^{\infty} f(x,t)\, e^{-ux-vt}\, dxdt. \tag{4}$$

The double Mohand transform operator is the name given to the operator M_2 in this regard. The definition of the inverse double Mohand transform is

$$M_2^{-1}[R(u,v)] = f(x,t). \tag{5}$$

2.2 Properties of DMT

The fundamental properties of DMT are given in the following table 1 [42], and table 2 for some basic functions of double Mohand transform and their inverse transforms are given by [42].

Table 1. Fundamental properties of DMT

S. No	Name of property	Mathematical form
1	Linearity	$M_2[\alpha f(x,t) + \beta g(x,t) = \alpha R_1(u,v) + \beta R_2(u,v)$
2	Change of scale	$M_2[f(ax,bt)] = abR\left(\frac{u}{a},\frac{v}{b}\right)$
3	Shifting	$M_2[e^{ax+bt}f(x,t)] = \frac{u^2}{(u-a)^2}\frac{v^2}{(v-b)^2}R(u-a,v-b)$
4	First Derivative regarding x and t	$M_2\left[\frac{\partial}{\partial x}f(x,t)\right] = uR(u,v) - u^2R(0,v)$ $M_2\left[\frac{\partial}{\partial t}f(x,t)\right] = vR(u,v) - v^2R(u,0)$
5	Second Derivative regarding x and t	$M_2\left[\frac{\partial^2}{\partial x^2}f(x,t)\right] = u^2R(u,v) - u^3R(0,v) - u^2\frac{\partial}{\partial x}R(0,v)$ $M_2\left[\frac{\partial^2}{\partial t^2}f(x,t)\right] = v^2R(u,v) - v^3R(u,0) - v^2\frac{\partial}{\partial t}R(u,0)$
6	nth Derivative regarding x and t	$M_2\left[\frac{\partial^n}{\partial x^n}f(x,t)\right] = u^nR(u,v) - u^{n+1}R(0,v) - u^n\frac{\partial}{\partial x}R(0,v) - \cdots - u^2\frac{\partial^{n-1}}{\partial x^{n-1}}R(0,v)$ $M_2\left[\frac{\partial^n}{\partial t^n}f(x,t)\right] = v^nR(u,v) - v^{n+1}R(u,0) - v^n\frac{\partial}{\partial t}R(u,0) - \cdots - v^2\frac{\partial^{n-1}}{\partial t^{n-1}}R(u,0)$
7	Convolution	$M_2[f_1(x,t) * f_2(x,t) = \frac{1}{u^2}\frac{1}{v^2}R_1(u,v)R_2(u,v)$

3. BASIC IDEA OF DMT TECHNIQUE

In this section of the current work, we explain the DMT technique and demonstrate how it can be employed PDEs in order to solve them. The wave equation, in its generic form, is given by

$$f_{tt}(x,t) = \alpha f_{xx}(x,t) + \beta f(x,t) + \omega(x,t)\,, \tag{6}$$

along with suitable initial settings

$$f(x,0) = a_1(x), \quad f_t(x,0) = a_2(x), \tag{7a}$$

and appropriate boundary settings

$$f(0,t) = b_1(t), \qquad f_x(0,t) = b_2(t), \tag{7b}$$

where $\omega(x,t)$ is the source term, α and β are constants, and $f(x,t)$ is an unknown function. For certain applications, an explicit method for the solution of a differential equation is formulated. The principal idea of this technique is to apply the single Mohand transform to the initial and boundary settings, and the double Mohand transform to Eq. (6).The initial condition is subjected to the single Mohand transform as follows

$$M_1[f(x,0)] = M_1[a_1(x)] = A_1 = R(u,0),$$
$$M_1[f_t(x,0)] = M_1[a_2(x)] = A_2 = \frac{\partial}{\partial t}R(u,0). \tag{8}$$

Table 2. Some basic functions of double Mohand transform and their inverse transforms [42]

S. No	$f(x,t)$	$M_2[f(x,t)]$	$M_2^{-1}[R(u,v)] = f(x,t)$
1	1	uv	1
2	xt	1	Xt
3	x^2t^2	$\frac{2!}{uv}$	x^2t^2
4	$x^nt^m; m,n \in N$	$\frac{n!\,m!}{u^{n-1}v^{m-1}}$	x^nt^m
5	$x^nt^m; m,n > -1$	$\frac{\Gamma(n+1)\Gamma(m+1)}{u^{n-1}v^{m-1}}$	x^nt^m
6	e^{ax+bt}	$\frac{u^2}{u-a}\frac{v^2}{v-b}$	e^{ax+bt}
7	$\sin(ax)\sin(bt)$	$\frac{au^2}{u^2+a^2}\frac{bv^2}{v^2+b^2}$	sinaxsinbt
8	$\cos(ax)\cos(bt)$	$\frac{u^3}{u^2+a^2}\frac{bv^3}{v^2+b^2}$	cosaxcosbt
9	$\sinh(ax)\sinh(bt)$	$\frac{au^2}{u^2-a^2}\frac{bv^2}{v^2-b^2}$	sinhaxsinhbt
10	$\cosh(ax)\cosh(bt)$	$\frac{u^3}{u^2-a^2}\frac{bv^3}{v^2-b^2}$	coshaxcoshbt

Applying single Mohand transform to the boundary conditions as

$$M_1[f(0,t)] = M_1[b_1(t)] = B_1 = R(0,v), \tag{9}$$

$$M_1[f_x(0,t)] = M_1[b_2(t)] = B_2 = \frac{\partial}{\partial x}R(0,v). \tag{10}$$

The application of the DMT to both sides of Eq. (6) results in

$$M_2[f_{tt}(x,t)] = M_2[\alpha f_{xx}(x,t) + \beta f(x,t) + \omega(x,t)].$$

With the aforementioned modified conditions and the differential properties of DMT, we have

$$v^2R(u,v) - v^3R(u,0) - v^2\frac{\partial}{\partial t}R(u,0)$$
$$= \alpha\left[u^2R(u,v) - u^3R(0,v) - u^2\frac{\partial}{\partial x}R(0,v)\right] + \beta R(u,v) + M_2[\omega(x,t)],$$
$$v^2R(u,v) - v^3A_1 - v^2A_2 = \alpha(u^2R(u,v) - u^3B_1 - u^2B_2) + \beta R(u,v) + M_2[\omega(x,t)]. \quad (11)$$

Eq. (11) can also be written as

$$R(u,v) = \frac{v^3A_1 + v^2A_2 - \alpha u^3B_1 - \alpha u^2B_2 + M_2[\omega(x,t)]}{v^2 - \alpha u^2 - \beta}. \quad (12)$$

The application of the inverse DMT to both sides of Eq. (11) results in

$$f(x,t) = M_2^{-1}\left[\frac{v^3A_1 + v^2A_2 - \alpha u^3B_1 - \alpha u^2B_2 + M_2[\omega(x,t)]}{v^2 - \alpha u^2 - \beta}\right], \quad (13)$$

where the term denoted by $f(x,t)$ is the result of the initial and boundary conditions, as well as the known function $\omega(x,t)$. $f(x,t)$ gives the exact solution of given wave equation.

4. APPLICATIONS OF DMT TO SOLVE WAVE EQUATIONS:

In this part of the article, we employ the DMT to solve some problems on wave equations.

Example 1: Consider the wave equation [27]

$$f_{tt} = c^2f_{xx}, \quad (14)$$

along with the appropriate initial and boundary settings

$$f(x,0) = sinx\,;\ f_t\,(x,0) = 2,$$
$$f(0,t) = 2t\,;\ f_x\,(0,t) = \cos ct.$$

Solution:

Applying single Mohand transform to initial and boundary conditions we have

$$M[f(x,0)] = M[sinx] = \frac{u^2}{u^2+1} = (u,0) = A_1,$$
$$M[f_t(x,o)] = M[2] = 2M[1] = 2u = \frac{\partial}{\partial t}R(v,0) = A_2,$$
$$M[u(0,t)] = 2M[t] = 2 = R\,(0,v) = B_1,$$
$$M[f_x(0,t)] = M[cos\ ct] = \frac{v^3}{v^2 + c^2}\frac{\partial}{\partial x}\,R\,(0,v) = B_2.$$

By putting the values of $\alpha = c^2$, A_1, A_2, B_1, B_2, $\beta = 0$ and $\omega(x,t) = 0$ in the generic formula in Eq. 12, we get

$$R(u,v) = \frac{\frac{u^2}{u^2+1}v^3 + 2uv^2 - 2c^2u^3 - c^2u^2\frac{v^3}{v^2+c^2}}{v^2 - c^2u^2},$$

$$= \frac{v^3u^2}{(u^2+1)(v^2+c^2)} + 2u. \tag{15}$$

The solution to Eq. (14) is attained by applying the inverse double Mohand transform to Eq. (15), resulting in

$$f(x,t) = sinxcosct + 2t.$$

Figure 1 shows the three-dimensional graph representing the exact solution of Example 1.

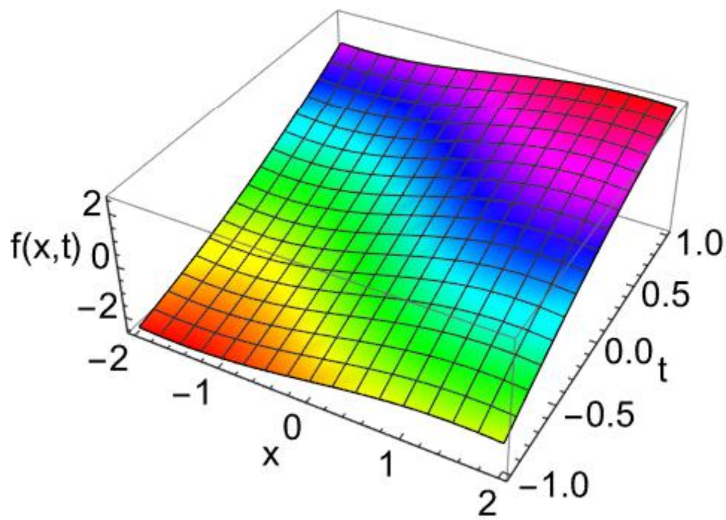

Figure 1. Three-dimensional plot of the exact solution for Example 1.

Example 2:

Let us consider the wave equation given by [27]

$$f_{tt} = f_{xx} - 3f + 3, \tag{16}$$

subject to the following initial and boundary conditions

$$f(x,0) = 1;\ f_t(x,0) = 2sinx,$$

$$f(0,t) = 1, f_x(0,t) = sin2t.$$

Solution:

The application of the single Mohand integral transform to the initial and boundary conditions yields

$$M[f(x,0)] = M[1] = u = R(u,0) = A_1,$$

$$M[f(x,0)] = 2M[sin2t] = \frac{2u^2}{u^2+1} = \frac{\partial R(u,0)}{\partial t} = A_2,$$

$$M[f(0,t)] = M[1] = v = R(0,v) = B_1,$$

$$M[f_x(0,t)] = M[sin2t] = \frac{2v^2}{v^2+4} = \frac{\partial R(0,v)}{\partial x} = B_2.$$

By putting the values of $\alpha = 1, A_1, A_2, B_1, B_2, \beta = 1$ and $\omega(x,t) = 3$ in the generic formula in Eq. 12, we get

$$R(u,v) = \frac{v^3 u + \frac{2u^2}{u^2+1} v^2 - vu^3 - \frac{2v^2}{v^2+4} u^2 + M_2[3]}{v^2 - \alpha u^2 - \beta},$$

$$= \left[uv + \left(\frac{u^2}{u^2+1}\right)\left(\frac{2u^2}{v^2+4}\right)\right]. \tag{17}$$

The solution to Eq. (16) is obtained by applying the inverse double Mohand transform to Eq. (17), resulting in

$$f(x,t) = 1 + \sin x \, sin2t.$$

The 3D graph of the exact solution to Example 2 is depicted in Figure 2.

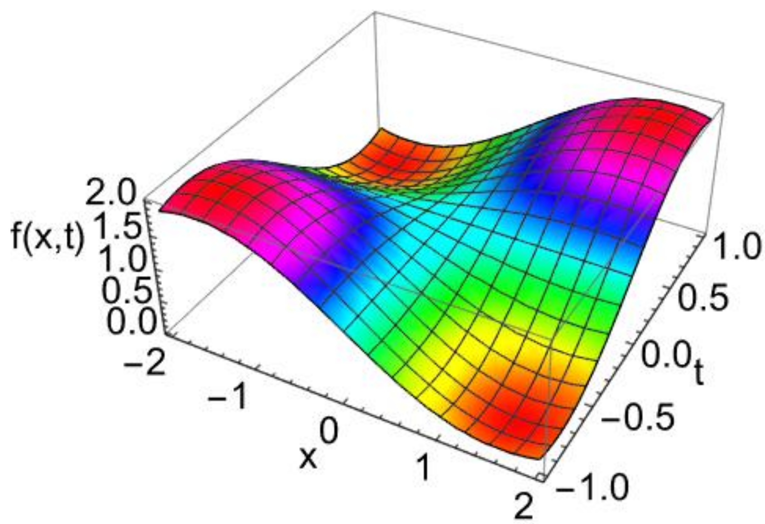

Figure 2. 3D visualization of the exact solution for Example 2.

Example 3:

The wave equation under attention is [43]

$$f_{tt} = f_{xx}, \tag{18}$$

along with its initial and boundary settings

$$f(x,0) = sinx; \qquad f_t\,(x,0) = 0,$$

$$f(0,t) = 0; \qquad f_x(0,t) = cost.$$

Solution:

The initial and boundary settings are subjected to the single Mohand transform, which results in

$$M[sinx] = \frac{u^2}{u^2+1} = R\,(u,0) = A_1,$$

$$M[f_t(x,0)] = M_2[0] = 0 = \frac{\partial}{\partial t} R\,(u,0) = A_2,$$

$$M[f(0,t)] = 0 = R\,(0,v) = B_1$$

$$M[f_x\,(0,t)] = M_2[cost] = \frac{v^3}{v^2+1} = \frac{\partial}{\partial x} R(0,v) = B_2.$$

By putting the values of $\alpha = 1$, A_1, A_2, B_1, B_2, $\beta = 0$ and $\omega(x,t) = 0$ in the generic formula in Eq. 12, we get

$$R(u,v) = \frac{v^3 \dfrac{u^2}{u^2+1} - u^2 \dfrac{v^3}{v^2+1}}{v^2 - u^2} = \frac{u^2}{u^2+1}\frac{v^3}{v^2+1}. \qquad (19)$$

The solution to Eq. (18) is attained by applying the inverse double Mohand integral transform to Eq. (19)

$$f(x,t) = sinxcost.$$

Figure 3 shows the three-dimensional graph depicting the exact solution for Example 3.

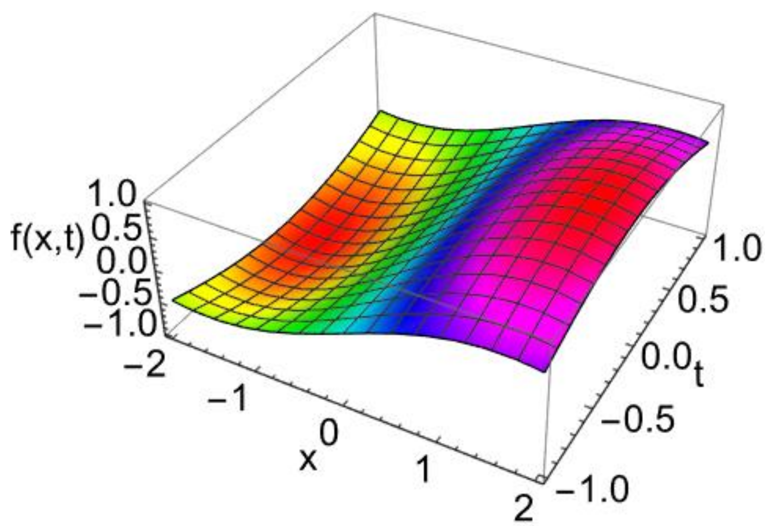

Figure 3. 3D visualization of the exact solution for Example 3.

Example 4:

The wave equation of attention is given as follows [43]

$$f_{tt} = f_{xx}, \qquad (20)$$

subject to the following initial and boundary settings

$$f(x,0) = 0; \quad f_t(x,0) = cosx,$$

$$f(0,t) = sinht; \quad f_x(0,t) = 0.$$

Solution:

The application of the single Mohand integral transform to the initial and boundary settings gives

$$M[0] = 0 = R(u,0) = A_1,$$

$$M[f_t(x,0)] = M[cosx] = \frac{u^3}{u^2+1} = \frac{\partial R(u,0)}{\partial t} = A_2,$$

$$M[f(0,t)] = M[sinht] = \frac{v^2}{v^2-1} = R(0,v) = B_1,$$

$$M[f_x(0,t)] = M[0] = 0 = \frac{\partial R(0,v)}{\partial x} = B_2.$$

By putting the values of $\alpha = -1, A_1, A_2, B_1, B_2, \beta = 0$ and $\omega(x,t) = 0$ in the generic formula in Eq. 12, we get

$$R(u,v) = \frac{v^2 \frac{u^3}{u^2+1} + u^3 \frac{v^2}{v^2-1}}{v^2+u^2} = \frac{u^3}{(u^2+1)} \frac{v^2}{(v^2-1)}. \tag{21}$$

The solution to Eq. (20) is attained by applying the inverse double Mohand integral transform to Eq. (21)

$$f(x,t) = cosx\ sinht.$$

Figure 4 portrays the three-dimensional plot of the exact solution for Example 4.

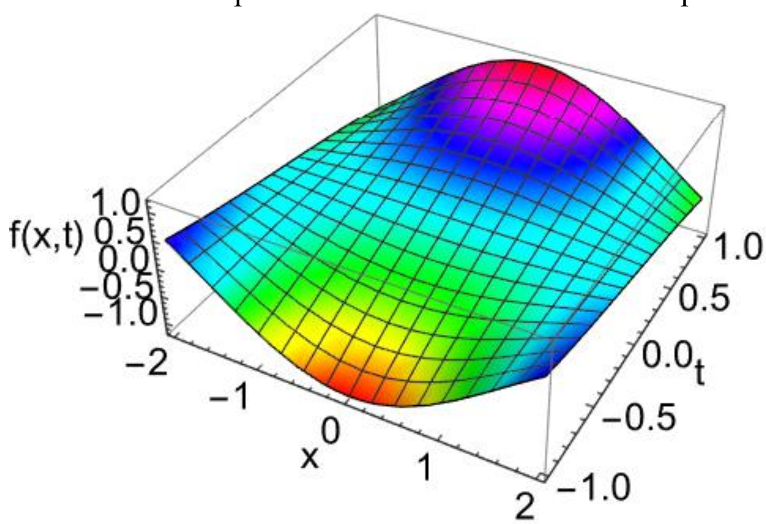

Figure 4. The graph of exact solution of Example 4.

Example 5:

We consider a wave equation of the following form [44]

$$f_{tt} - f_{xx} = 6t + 2x,$$

$$f_{tt} = f_{xx} + 6t + 2x, \tag{22}$$

together with its following associated initial and boundary settings

$$f(x,0) = 0; \quad f_t(x,0) = sinx,$$

$$f(0,t) = t^3; \quad f_t(0,t) = t^2 + sint,$$

Solution:

The application of the single Mohand integral transform to the given initial and boundary settings leads to

$$M[f(x,0)] = M[0] = 0 = A_1,$$

$$M[f_t(x,0)] = M[sinx] = \frac{u^2}{u^2+1} = A_2,$$

$$M[f(0,t)] = M[t^3] = \frac{6}{v^2} = B_1,$$

$$M[f_x(0,t)] = M[t^2 + sint] = M[t^2] + M[sint] = \frac{2}{v} + \frac{v^2}{v^2+1} = B_2.$$

By putting the values of $\alpha = 1, A_1, A_2, B_1, B_2, \beta = 0$ and $\omega(x,t) = 6t + 2x$ in the generic formula in Eq. 12, we get

$$R(u,v) = \frac{v^2 \frac{u^2}{u^2+1} - u^3 \frac{6}{v^2} - u^2\left(\frac{2}{v} + \frac{v^2}{v^2+1}\right) + M_2[6t+2x]}{v^2 - u^2},$$

$$= \frac{v^2 \frac{u^2}{u^2+1} - u^3 \frac{6}{v^2} - u^2\left(\frac{2}{v} + \frac{v^2}{v^2+1}\right) + 6u + 2v}{v^2 - u^2},$$

$$= \frac{u^2}{u^2+1} \cdot \frac{v^2}{v^2+1} + \left(\frac{6u+2v}{v^2}\right), \tag{23}$$

By applying the inverse double Mohand integral transform to Eq. (23), we get the solution to Eq. (22)

$$f(x,t) = sinxsinht + xt^2 + t^3.$$

The 3D plot of the exact solution for Example 5 is portrayed in Figure 5.

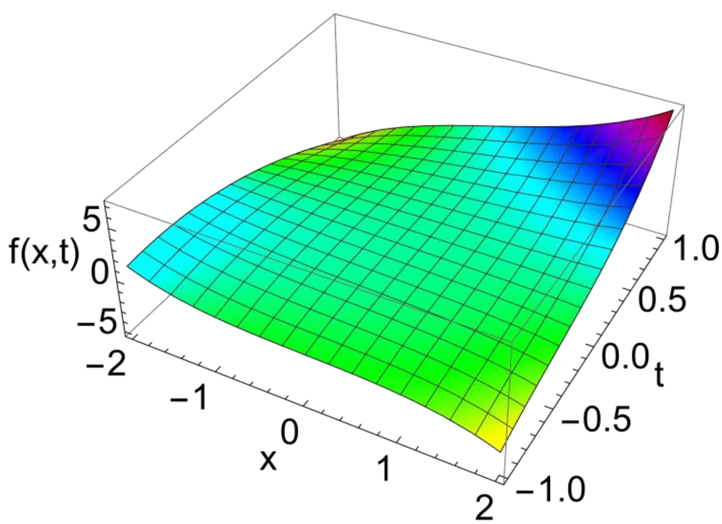

Figure 5. Three-dimensional plot of the exact solution for Example 5.

5. CONCLUSION:

This article represents the application of the recently introduced double Mohand integral transform, complemented by a set of theorems and properties that highlight its potential and establish its theoretical foundation. Finally, DMT was applied in the solution of some wave equation applications. A remarkably efficient technique for wave equations was formulated and implemented for several cases. Each of these cases illustrated how remarkably simple and effective DMT is for solving partial differential equations. Thus, the ease with which further solutions can be obtained encourages using DMT in the future, especially considering how advantageous it is to retain constant values during the transform, allowing greater efficiency in comparison to other integral transforms. The initial problem formulated in one dimension can be generalized into two and three dimensions for further research. Additionally, the nonlinear wave equation in all three dimensions is of interest in many physical problems with the goal to study its solutions.

Author Contribution

- Conceptualization, Data curation: M. Abbas
- Formal analysis, Validation: S. Ali
- Writing - original draft: W. Rehman
- Writing - review editing: S. Ullah and W. Rehman

Conflicts of Interest

The author declares no conflicts of interest.

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Data Availability Statement

The generated datasets used and evaluated in this research study are provided in the form of graphs and tables within the manuscript.

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