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Research Article

Solving Heat Equations via Double Mohand Transform

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ABSTRACT

Partial Differential Equations are vital in applied mathematics, chemistry, and physics since they model a variety of phenomena including the dynamics of fluids, the conduction of heat, and the propagation of waves. Due to their wide-ranging applications, PDEs have received a lot of attention for the development of efficient and accurate solution methodologies. Most of these attempts stress on accomplishing symmetry that simplifies the equations and facilitates easier solutions, whether analytical or numerical, to be found. To offer a different and efficient method to solving PDEs, we introduce in this article the Double Mohand Transform Method, an innovative integral transform method. This innovative double transform technique builds upon former integral transforms by incorporating vital elements that enhance solution efficiency and applicability. In this work, we explain comprehensively the double Mohand transform with special focus on its defining features, advantages, and unique qualities. In a more theoretical context, we present new results regarding its foundations and the partial derivatives of the transform. To explore the importance as well as effectiveness of the Double Mohand Transform Technique, we apply it on some cases of the heat equation with different boundary and initial conditions. By exploiting the symmetry features of the heat equation, our method correctly deals with complex PDEs by providing elegant and efficient solutions. The outcomes illustrate how this novel transformation could further progress the mathematical methods for differential equations applicable in various branches of science and engineering.

1. INTRODUCTION

Engineering and Scientific applications are frequently dependent on the solution of differential equations (DEs) subject to established boundary and initial conditions. These conditions define the bounds within which a solution is computed, hence they are vital for the valid representation of physical processes. In case of ODEs, we usually have a problem of recognizing the specified conditions to begin with, and then changing the constants to fulfill those conditions. This systematic approach always works for finding solutions which comply with the particular limits. But this procedure cannot be directly employed when solving partial differential equations (PDEs). Unlike ODEs, PDEs are governed by a multitude of independent variables. Thus, the problem of applying the initial and boundary conditions is far more difficult with the rest of the system conditions. Frequently, the interplay between different terms in a PDE requires sophisticated analytical or numerical techniques to achieve compliance with required conditions

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based on the solution. The reason for this complexity is that PDEs typically describe dynamic systems that involve multiple spatial and temporal variables that are interconnected, for example, in fluid dynamics, electromagnetism, and quantum mechanics.

Boundary [1–3] and initial value problems [4-6] are two key types of differential equations from the domain of mathematics which are used to model real world systems. Moreover, basic equations like the heat equation, wave equation, and other PDE based formulations [7-10] are frequently employed to describe physical phenomena including heat conduction, vibration, and wave propagation. Due to the complexities associated with these problems, they are often solved through the Hirota bilinear method, separation of variables, Fourier series, numerical discretization, and other higher approaches. With these techniques, scientists and engineers are able to develop solutions within the specified bounds and precisely estimate complex systems enabling detailed analysis.

There are many analytical and numerical approaches to solve partial differential equations (PDEs). Each technique differs based on the type of problem. Analytical techniques include the separation of variables technique which breaks the PDE into simpler sub-ODEs and algebraic approaches using ARA, Laplace, Fourier, and double transforms (e.g., double Laplace-Sumudu [16] and double ARA-Sumudu [17]) that alter the form of the PDE to algebraic like format [18-20]. The Adomian decomposition technique is suitable for providing series solutions to non-linear PDEs, while the convolution method is helpful in integral limited systems [11-15]. Numerical approaches like the finite difference approach (FDA), finite element approach (FEA), and the finite volume approach (FVA) are used to get approximate solutions when analytical solutions are not practicable. However, spectral and meshless approaches provide better accuracy, especially for complicated and irregular computational realms. These approaches allow for successful modeling of heat conduction, wave propagation, and fluid dynamics in both engineering and science.

Integral transformations make differential, integral, and system of equations easier to understand and solve. The improper integral given below may be calculated to obtain the integral transform of the function $g(t)$ where $t \in (-\infty, \infty)$

$$L[g(t)](s) = \int_{-\infty}^{\infty} h(s,t)\, g(t) dt, \tag{1}$$

Where s can be either real or complex, and $h(s,t)$ represents the kernel of transform, a function of two variables that is unaffected by the factor t.

The accomplishment of integral transforms in solving engineering and physical problems has led to the development of numerous transforms, attracting significant attention from applied mathematicians. A few scenarios are the Mellin integral transform [21], Hankel's integral transform [22], the Laplace-Carson integral transform [23], the Sumudu integral transform [24], and many others [25,26]. Double integral transforms substantially assist problem-solving by transforming ordinary and partial differential expressions into equivalent algebraic forms. Single integral transformations are more successful in solving a variety of physical problems when united with other methods.

The Mohand integral transform, which was first put forward by the authors of [27] in 2017, is a valuable tool for handling ordinary and partial differential equations in addition to integral equations. In addition to suggesting new double Mohand transform (DMT) approach, the authors [28] offer the basic ideas,

features, as well as particular uses of the DMT in solving physical and engineering problems. Solutions to five heat equation examples are given in current study. By means of simple mathematical operations, the double transform suggested in the present investigation transforms functions of two factors into functions of four factors. Furthermore, the explicit duality between the formable transform and the Laplace transform is demonstrated via the utilization of a straightforward replacement and a scalar multiplicative factor. This attribute makes it practicable to solve partial differential equations with the recommended method, presenting the benefit of less computational work and retaining fixed values under the DMT.
The following form of the nonhomogeneous linear heat equation is investigated in the present work

$$f_t\,(x,t) = \alpha f_{xx}(x,t) + \beta f(x,t) + g(x,t)\,, \tag{2}$$

with initial condition

$$f(x,t) = c_1 x\,, \tag{3}$$

and the boundary conditions

$$f(0,t) = d_1(t)\,, \qquad f_x(0,t) = d_2(t)\,, \tag{4}$$

where $g(x,t)$ is the source term, α and β are constants, and $f\,(x,t)$ is unknown function.
A simple formula that has been developed and employed in numerous scenarios can be employed to solve the mentioned earlier equation. The recommended formula utilizes alternative transformations to handle heat-type PDEs and demands less computational work than traditional numerical approaches. The following is how this paper is structured: This article's structure is arranged as follows: Section 1 introduces the investigation; Section 2 discusses the key ideas as well as features of the new double Mohand transform; Section 3 provides the basic framework for employing the DMT to solve the heat equation as well as find the solution formula; Section 4 gives and solves a number of illustrative examples using the DMT; and Section 5 encompasses up the study's contents.

2. DOUBLE MOHAND TRANSFORM (DMT)

2.1 Definition

The double Mohand transform of function f is defined as follows for any values of *x* and *t* greater or equal to 0 [28]

$$M_2[f(x,t){:}\,u,v\,] = R\,(u,v) = u^2v^2 \iint_0^\infty f(x,t)\,e^{-ux-vt}\,dxdt. \tag{5}$$

In this context, the operator M_2 is referred to as the double Mohand transform operator. The inverse double Mohand transform is defined as

$$M_2^{-1}\,[R(u,v)] = f\,(x,t). \tag{6}$$

2.2 Properties of DMT

Table 1 below provides the basic characteristics of DMT [28], while Table 2 provides several elementary functions of the double Mohand transform and their inverse transforms [28].

Table 1. Basic characteristics of DMT

S. No	Name of property	Mathematical form
1	Linearity	$M_2[\alpha f(x,t) + \beta g(x,t) = \alpha R_1(u,v) + \beta R_2(u,v)$
2	Change of scale	$M_2[f(ax,bt)] = abR(\frac{u}{a},\frac{v}{b})$
3	Shifting	$M_2[e^{ax+bt}f(x,t)] = \frac{u^2}{(u-a)^2}\frac{v^2}{(v-b)^2}R(u-a,v-b)$
4	First Derivative	$M_2\left[\frac{\partial}{\partial x}f(x,t)\right] = uR(u,v) - u^2R(0,v)$
5	Second Derivative	$M_2\left[\frac{\partial^2}{\partial x^2}f(x,t)\right] = u^2R(u,v) - u^3R(0,v) - u^2\frac{\partial}{\partial x}R(0,v)$
6	nth Derivative	$M_2\left[\frac{\partial^n}{\partial x^n}f(x,t)\right] = u^nR(u,v) - u^{n+1}R(0,v) - u^n\frac{\partial}{\partial x}R(0,v) - \cdots - u^2\frac{\partial^{n-1}}{\partial x^{n-1}}R(0,v)$
7	Convolution	$M_2[f_1(x,t) * f_2(x,t) = \frac{1}{u^2}\frac{1}{v^2}R_1(u,v)R_2(u,v)$

Table 1. Elementary functions of the DMT and their inverse transforms

S. No	$f(x,t)$	$M_2[f(x,t)$	$M_2^{-1}[R(u,v)] = f(x,t)$
1	1	uv	1
2	xt	1	xt
3	x^2t^2	$\frac{2!}{uv}$	x^2t^2
4	$x^nt^m; m,n \in N$	$\frac{n!\,m!}{u^{n-1}v^{m-1}}$	x^nt^m
5	$x^nt^m; m,n > -1$	$\frac{\Gamma(n+1)\Gamma(m+1)}{u^{n-1}v^{m-1}}$	x^nt^m
6	e^{ax+bt}	$\frac{u^2}{u-a}\frac{v^2}{v-b}$	e^{ax+bt}
7	$\sin ax \sin bt$	$\frac{au^2}{u^2+a^2}\frac{bv^2}{v^2+b^2}$	$\sin ax \sin bt$
8	$\cos ax \cos bt$	$\frac{u^3}{u^2+a^2}\frac{bv^3}{v^2+b^2}$	$\cos ax \cos bt$
9	$\sinh ax \sinh bt$	$\frac{au^2}{u^2-a^2}\frac{bv^2}{v^2-b^2}$	$\sinh ax \sinh bt$
10	$\cosh ax \cosh bt$	$\frac{u^3}{u^2-a^2}\frac{bv^3}{v^2-b^2}$	$\cosh ax \cosh bt$

3. BASIC IDEA OF DMT TECHNIQUE

This section introduces the DMT procedure and its application to heat equations, signifying its usefulness in solving partial differential equations. The heat equation can be written in the following generic style

$$f_t(x,t) = \alpha f_{xx}(x,t) + \beta f(x,t) + g(x,t), \tag{7}$$

along with the initial and boundary conditions

$$f(x,0) = c_1(x), \tag{8}$$

$$f(0,t) = d_1(t), \qquad f_x(0,t) = d_2(t), \tag{9}$$

where $g(x,t)$ is the source term, α and β are constants, and $f(x,t)$ is an unknown function. To efficiently solve differential equations in various applications, a simple formula is developed. This approach leverages the single Mohand transform for initial and boundary conditions while applying the double Mohand transform to Eq. (7). The initial condition is subjected to the single Mohand integral transform in the subsequent way

$$M_1[f(x,0)] = M_1[c_1(x)] = A_1 = R(u,0).$$

Applying single Mohand transform to the boundary conditions as

$$M_1[f(0,t)] = M_1[d_1(t)] = B_1 = R(0,v),$$

$$M_1[f_x(0,t)] = M_1[d_2(t)] = B_2 = \frac{\partial}{\partial x}R(0,v).$$

The DMT can now be applied to both sides of Eq. (6) to get

$$M_2[f_t(x,t)] = M_2[\alpha f_{xx}(x,t) + \beta f(x,t) + g(x,t)].$$

Using the differential characteristics of DMT and the previously indicated modifications, we have

$$vR(u,v) - u^2R(u,0) = \alpha\left[u^2R(u,v) - u^3R(0,v) - u^2\frac{\partial}{\partial x}R(0,v)\right] + \beta R(u,v) + M_2[g(x,t)],$$

$$vR(u,v) - v^2A_1 = \alpha(u^2R(u,v) - u^3B_1 - u^2B_2) + \beta R(u,v) + M_2[g(x,t)]. \tag{10}$$

Eq. (10) can be rearranged as

$$R(u,v) = \frac{v^2A_1 - \alpha u^3B_1 - \alpha u^2B_2 + M_2[g(x,t)]}{v - \alpha u^2 - \beta}. \tag{11}$$

Both sides of expression (11), when subjected to the inverse double Mohand transform (DMT), result

$$f(x,t) = M_2^{-1}\left[\frac{v^2A_1 - \alpha u^3B_1 - \alpha u^2B_2 + M_2[g(x,t)]}{v - \alpha u^2 - \beta}\right], \tag{12}$$

where the initial and boundary conditions, along with the established function $g(x,t)$, give rise to the function $f(x,t)$.

4. APPLICATIONS OF DMT TO SOLVE HEAT EQUATIONS

Now, we solve a few heat equation problems employing the double Mohand integral transform approach.

Example 1. Consider the heat equation of the form [29]

$$f_t(x,t) = f_{xx}(x,t); \qquad x,t \geq 0, \tag{13}$$

subject to initial

$$f(x,0) = sinx \text{ and } f(0,t) = 0,$$

and appropriate boundary conditions

$$f_x(0,t) = e^{-t}.$$

Solution. Employing the initial and boundary conditions and the single Mohand integral transform technique, we obtain

$$A_1 = \frac{u^2}{u^2+1} \quad , \quad B_1 = 0 \quad , \quad B_2 = \frac{v^2}{v+1}, \tag{14}$$

If we compare Eq. (13) with Eq. (7), we see that $\alpha = 1, \beta = 0$ and $g(x,t) = 0$. By using Eq. (12), Eq. (13) can be written as

$$f(x,t) = M_2^{-1}\left[\frac{v^2A_1 - u^3B_1 - u^2B_2}{v-u^2}\right],$$

$$f(x,t) = M_2^{-1}\left[\frac{v^2\frac{u^2}{u^2+1} - u^3(0) - u^2\frac{v^2}{v+1}}{v-u^2}\right],$$

$$= M_2^{-1}\left[u^2v^2\left(\frac{\frac{1}{u^2+1} - \frac{1}{v+1}}{v-u^2}\right)\right].$$

After simplification, we have

$$f(x,t) = M_2^{-1}\left[\frac{v^2}{v+1}\frac{u^2}{u^2+1}\right] = e^{-t}sinx \tag{15}$$

Figure 1 depicts the three-dimensional exact solution for Example 1.

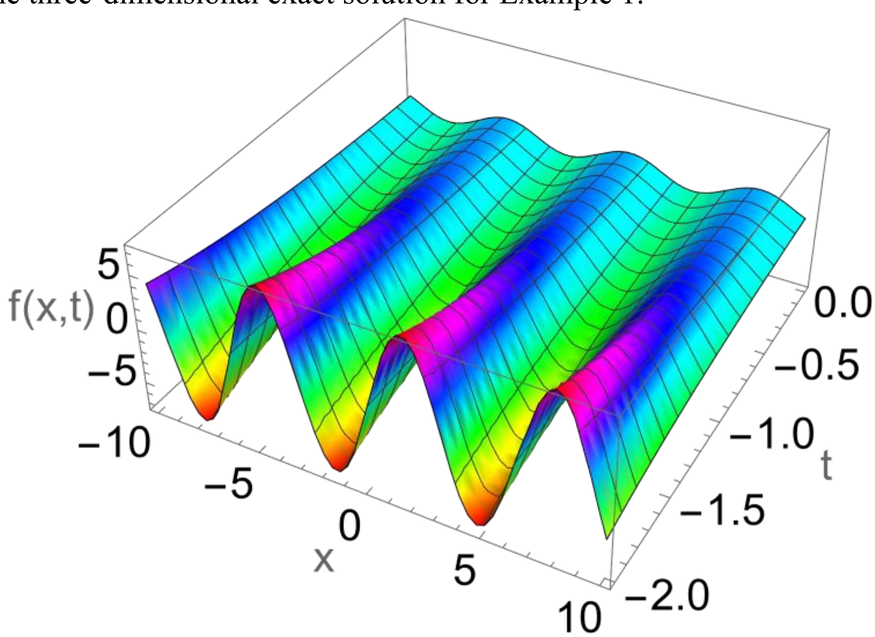

Figure 1. 3D plot of the exact solution corresponding to Example 1.

Example 2. The heat equation is expressed in the following form [29]

$$f_t(x,t) = 4f_{xx}(x,t)\,; \qquad x,t \geq 0, \tag{16}$$

along with its initial and appropriate boundary conditions $f(x,0) = cosx$ and $f(0,t) = e^{-4t}, f_x(0,t) = 0.$

Solution. Application of the single Mohand integral transform to the initial and appropriate boundary conditions results in

$$A_1 = \frac{u^3}{u^2+1} \quad , \quad B_1 = \frac{v^2}{v+4} \quad , \quad B_2 = 0, \tag{17}$$

If we compare Eq. (16) with Eq. (7), we say that $\alpha = 4, \beta = 0$, and $g(x,t) = 0$. By using Eq. (12), Eq. (16) can be written as

$$f(x,t) = M_2^{-1}\left[\frac{v^2A_1 - 4u^3B_1 - 4u^2B_2}{v-4u^2}\right],$$

$$= M_2^{-1}\left[\frac{v^2\frac{u^3}{u^2+1} - 4u^3\frac{v^2}{v+4} - 4u^2(0)}{v-4u^2}\right],$$

$$= M_2^{-1}\left[u^3v^2\left(\frac{\frac{1}{u^2+1} - \frac{1}{v+4}}{v-4u^2}\right)\right].$$

After simplification, we have

$$f(x,t) = M_2^{-1}\left[\frac{v^2}{v+4}\frac{u^3}{u^2+1}\right] = e^{-4t}cosx. \tag{18}$$

The three-dimensional exact solution of Example 2 is illustrated in Figure 2.

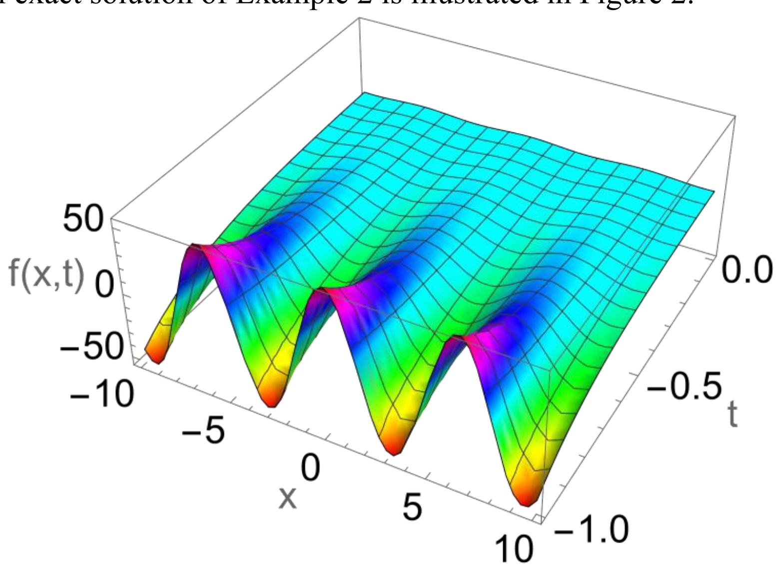

Figure 2. 3D plot of the exact solution corresponding to Example 2.

Example 3. The governing heat equation is formulated as [29]

$$f_t(x,t) = f_{xx}(x,t) - 2f(x,t); \qquad x,t \geq 0, \tag{19}$$

along with its initial and boundary conditions $f(x,0) = \sinh x$ and $f(0,t) = 0, f_x(0,t) = e^{-t}$.

Solution. By means of the single Mohand integral transform on the initial and boundary conditions, we get

$$A_1 = \frac{u^2}{u^2-1}, \qquad B_1 = 0 \quad , \quad B_2 = \frac{v^2}{v+1} \tag{20}$$

If we compare Eq. (19) with Eq. (7), we see that $\alpha = 1,\ \beta = -2$, and $g(x,t) = 0$. By using Eq. (12), Eq. (19) can be written as

$$f(x,t) = M_2^{-1}\left[\frac{v^2 A_1 - u^2 B_2}{v - u^2 + 2}\right],$$

$$= M_2^{-1}\left[\frac{v^2\left(\frac{u^2}{u^2-1}\right) - u^2\frac{v^2}{v+1}}{v - u^2 + 2}\right].$$

After simplification, we have

$$f(x,t) = M_2^{-1}\left[\frac{v^2}{v+1}\,\frac{u^2}{u^2-1}\right] = e^{-t}\sinh x \qquad (21)$$

Figure 3 provides a 3D plot of the exact solution corresponding to Example 3.

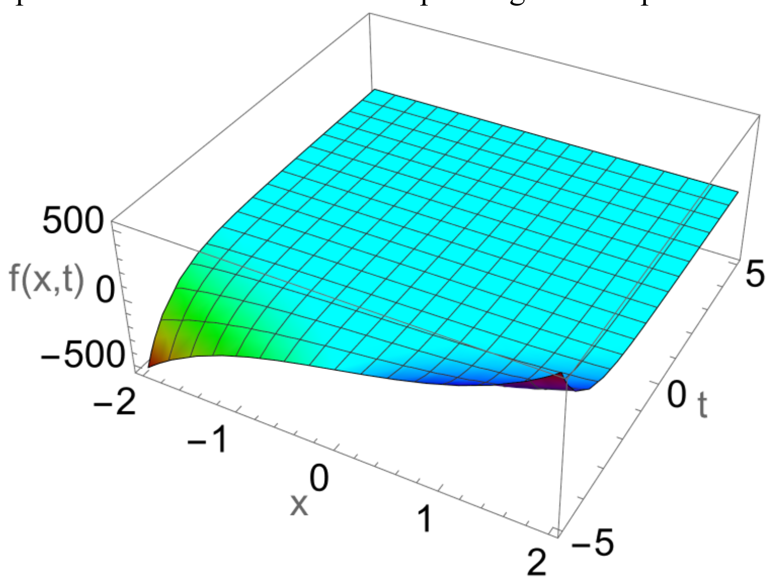

Figure 3. 3D plot of the exact solution corresponding to Example 3.

Example 4. The mathematical model for heat conduction is given by [29]

$$f_t(x,t) = f_{xx}(x,t) + \sin x; \qquad x,t \geq 0, \qquad (22)$$

with the corresponding initial and boundary conditions $f(x,0) = \cos x$ and $f(0,t) = e^{-t}, f_x(0,t) = 1 - e^{-t}$.

Solution. Upon employing the single Mohand integral transform to the initial and boundary conditions, we acquire

$$A_1 = \frac{u^3}{u^2+1}, \quad B_1 = \frac{v^2}{v+1}, \quad B_2 = v - \frac{v^2}{v+1}, \qquad (23)$$

If we compare eq. (22) with Eq. (7), we see that $\alpha = 1,\ \beta = 0$, and $g(x,t) = sinx$. By using Eq. (12), Eq. (13) can be written as

$$f(x,t) = M_2^{-1}\left[\frac{v^2 A_1 - u^3 B_1 - u^2 B_2 + M_2[\sin x]}{v - u^2}\right],$$

$$= M_2^{-1}\left[\frac{v^2\frac{u^3}{u^2+1}- u^3\frac{v^2}{v+1} - u^2\left(v-\frac{v^2}{v+1}\right)+v\frac{u^2}{u^2+1}}{v-u^2}\right].$$

After simplification, we have

$$f(x,t) = M_2^{-1}\left[\frac{v^2}{v+1}\frac{u^3}{u^2+1}+\left(v-\frac{v^2}{v+1}\right)\frac{u^2}{u^2+1}\right]$$
$$= e^{-t}\cos x + (1- e^{-t})\sin x \qquad (24)$$

The exact solution for Example 4, pictured in three dimensions, is depicted in Figure 4.

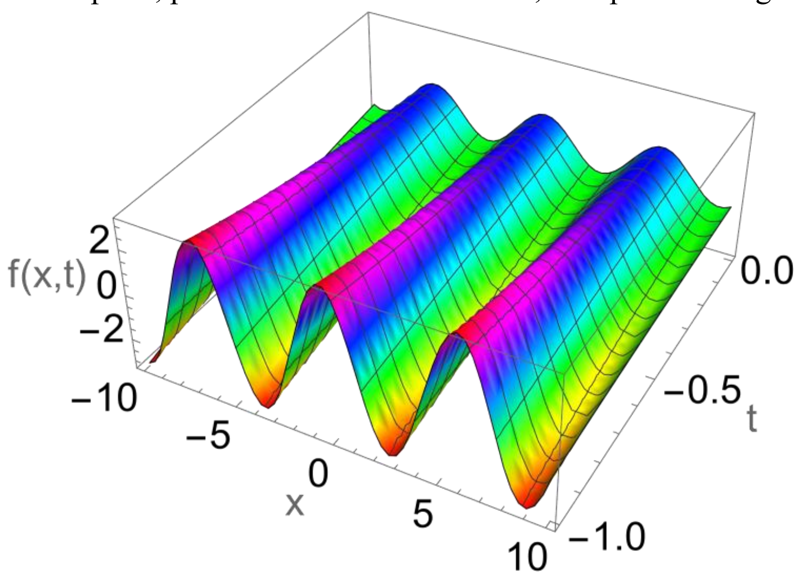

Figure 4. 3D plot of the exact solution corresponding to Example 4.

Example 5: Let us consider the heat equation stated as [29]

$$f_t(x,t) = f_{xx}(x,t) - 3f(x,t) + 3; \qquad x,t \geq 0, \qquad (25)$$

along with the specified initial and boundary conditions $f(x,0) = 1 + \sin x$ and $f(0,t) = 1, f_x(0,t) = e^{-4t}$.

Solution. When the single Mohand transform is subjected to the initial and boundary conditions, we obtain

$$A_1 = u + \frac{u^2}{u^2+1}, \qquad B_1 = v, \qquad B_2 = \frac{v^2}{v+4} \qquad (26)$$

If we compare Eq. (25) with Eq. (7), we see that $\alpha = 1,\ \beta = -3$, and $g(x,t) = 3$. By using Eq. (12), Eq. (25) can be written as

$$f(x,t) = M_2^{-1}\left[\frac{v^2A_1 - u^3B_1 - u^2B_2 + M_2[3]}{v-u^2+3}\right],$$

$$f(x,t) = M_2^{-1}\left[\frac{v^2\left(u+\frac{u^2}{u^2+1}\right) - u^3v - u^2\frac{v^2}{v+4}+3uv}{v-u^2+3}\right].$$

After simplification, we have

$$f(x,t) = M_2^{-1}\left[uv+\frac{v^2}{v+4}\frac{u^2}{u^2+1}\right] = 1+ e^{-4t}\sin x \qquad (27)$$

Figure 5 portrays the exact solution of Example 5 in three dimensions.

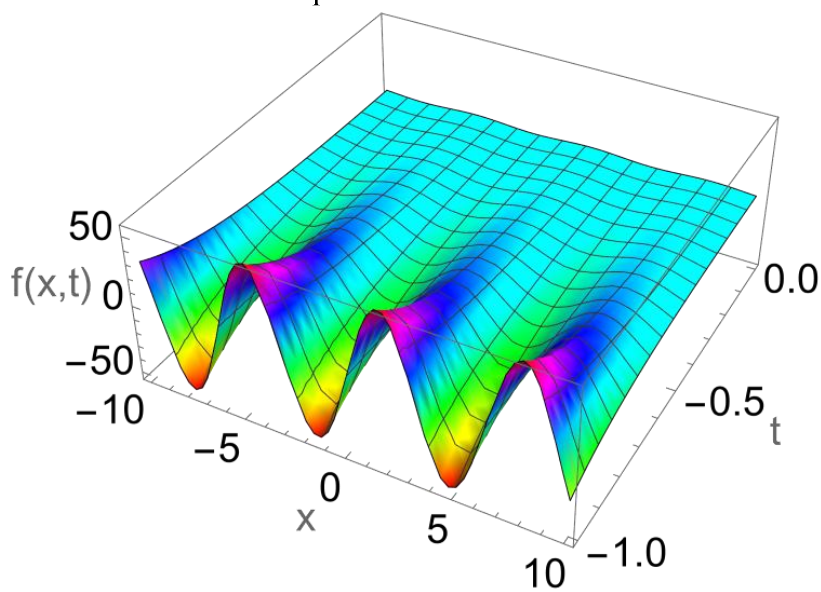

Figure 5. 3D plot of the exact solution corresponding to Example 5.

5. CONCLUSION

The definition, crucial features, and primary principles of the double Mohand transform (DMT) are discussed in this article, highlighting its application in solving heat equations. This article presents the definition, fundamental properties, and core principles of the double Mohand transform (DMT) as an effective tool for solving heat equations. We applied DMT to various heat equation problems, demonstrating its ability to simplify and efficiently solve partial differential equations (PDEs). A straightforward formula was established to obtain solutions, showcasing DMT's accuracy and effectiveness in handling complex PDEs. The study highlights the advantages of DMT, particularly its ability to maintain constant values under transformation while reducing computational effort compared to other integral transforms. Given its simplicity and computational efficiency, we aim to extend its application to a broader range of mathematical and engineering problems in the future.

Author Contribution

- Conceptualization: H. Haider
- Data curation: S. Ali
- Formal analysis: M. Abbas
- Validation: Shakir Ullah[1]
- Writing - original draft: W. Rehman
- Writing - review editing: W. Rehman

Conflicts of Interest

The author declares no conflicts of interest.

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Data Availability Statement

The generated datasets used and evaluated in this research study are provided in the form of graphs and tables within the manuscript.

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