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Research Article

Goodness-of-Fit Tests for Inverse Ishita Distribution and Its Application on Renewable Energy Data in the Kingdom of Saudi Arabia

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ABSTRACT

In this article, we presented the goodness-of-fit tests for the inverse Ishita distribution with one parameter based on complete sample. Moreover, the maximum likelihood estimation method was used to estimate the unknown parameter of the distribution. In addition, a Monte Carlo simulation was also performed to verify the behavior of the maximum likelihood estimator. Ultimately, we applied the proposed distribution on renewable energy data in Saudi Arabia and compared it with Weibull and Shanker distributions.

1. INTRODUCTION

Goodness-of-fit tests play an important role in determining whether the observed data fit the proposed models. Also, it is considered a useful tool in choosing the model that better fits the data among other models. For more details about the goodness of fit tests see [1].

In recent years, researchers have been interested in one-parameter distributions to find models better goodness of fit for data than the well-known distributions. Recently, one parameter distribution called "Ishita distribution" proposed by [2]. Some of its properties, parameter estimation and applications were investigated. In addition, there are many generalizations of Ishita distribution, such as the transmuted Ishita distribution proposed by [3]. Statistical and mathematical properties, estimate the parameter and application on real-life data were discussed. Moreover, [4] introduced the truncated Ishita distribution with some statistical properties and maximum likelihood estimation of its parameters. Shukla [5] proposed a new lifetime distribution that is an inverse of the Ishita distribution. Its statistical and mathematical properties were studied. In addition, the maximum likelihood estimator was used to estimate its parameter. Also, it was applied to lifetime data sets and compared with other inverse lifetime distributions. The CDF and PDF of inverse Ishita distribution with one parameter γ are given as follows:

$$F(x,\gamma) = \left(1 + \frac{\frac{\gamma}{x}\left(\frac{\gamma}{x}+2\right)}{\gamma^3+2}\right) e^{-\frac{\gamma}{x}} \quad ; x > 0\,, \gamma > 0, \tag{1}$$

$$f(x,\gamma) = \frac{\gamma^3}{x^4(\gamma^3+2)}(x^2\gamma + 1)e^{-\frac{\gamma}{x}} \quad ; x > 0\,, \gamma > 0. \tag{2}$$

where γ is a scale parameter. Through the following Fig 1 and Fig 2, we show the behavior for the curves of PDF, CDF for inverse Ishita distribution at several different values of the parameter.

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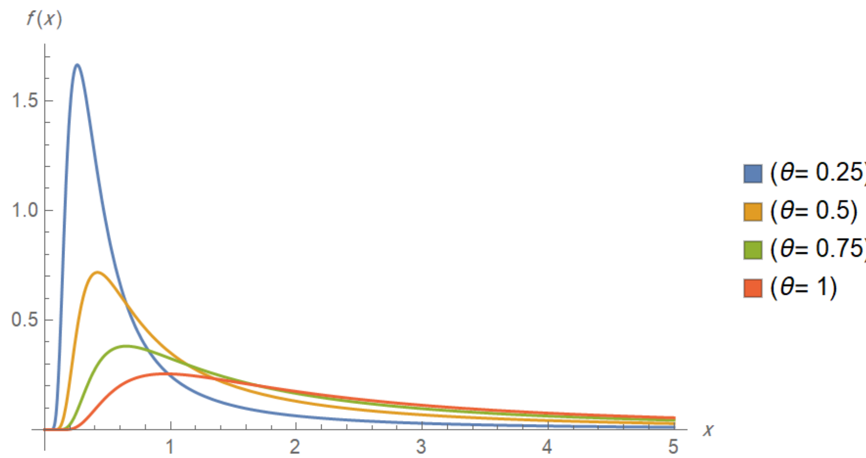

Figure 1. The PDF curves of the IID at $\theta = \{0.25, 0.5, 0.75, 1\}$.

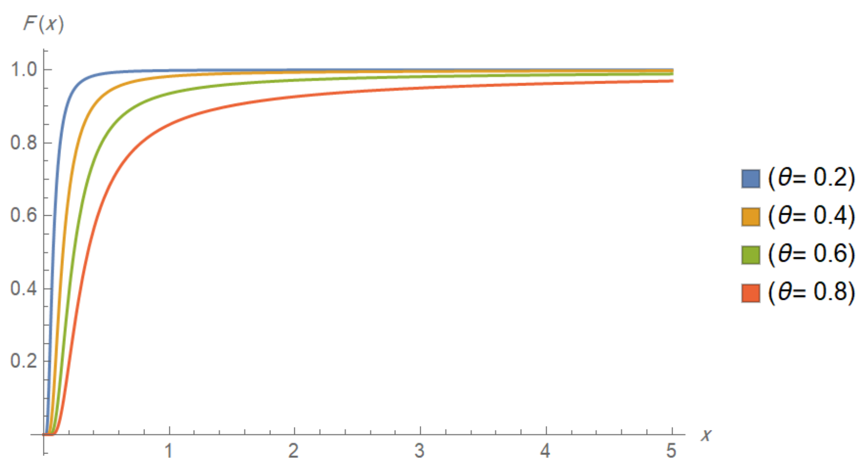

Figure 2. The CDF curves of the IID at $\theta = \{0.2, 0.4, 0.6, 0.8\}$.

On the other hand, different estimators of the shape parameter for exponentiated Gompertz distribution are studied in [6] and their performances were compared using numerical simulations. Furthermore, five methods of estimation for the Suja distribution parameter are considered in [7].
The aim of this paper is to estimate the parameter of inverse Ishita distribution by using the maximum likelihood estimation and perform numerical simulation at different sample sizes and different values of the parameter. Also, apply goodness of fit tests for the proposed distribution with other distributions to determine which one is the best fits for renewable energy data in Saudi Arabia.

2. MAXIMUM LIKELIHOOD ESTIMATION METHOD

The maximum likelihood estimator (MLE) of the inverse Ishita distribution for the parameter (γ) is derived in this section. Let $x_1, x_2, \dots, x_m$ be a random sample of size m from the inverse Ishita distribution, then the log-likelihood function is:

$$L = 3\,m\,ln(\gamma) - m\,ln(\gamma^3 + 2) + \sum_{i=1}^{m} ln(1 + \gamma x_i^2) - \sum_{i=1}^{m} ln({x_i}^4) - \gamma \sum_{i=1}^{m} \frac{1}{x_i}. \quad (4)$$

Taking the derivative of (4) with respect γ, and equating this to 0, we have:

$$\frac{3m}{\gamma}-\frac{3\gamma^2 m}{(\gamma^3+2)}+\sum_{i=1}^{m}\frac{x_i^2}{(1+\gamma\, x_i^2)}-\sum_{i=1}^{m}\frac{1}{x_i}=0 \qquad (5)$$

The maximum likelihood estimates of parameter γ can be found by solving the non-linear equation (5) using numerical methods like Newton Raphson.

3. NUMERICAL SIMULATION

In this section, we studied Monte Carlo simulations for different sample sizes to determine the efficiency of maximum likelihood estimate. The simulation is conducted by using Mathematica (V.12.1) and has been repeated 1000 times with different sample sizes $m = 30,50,100$ while choosing ($\gamma = 0.3,0.5,0.6$). The performance of the resulting estimator of the parameter has been considered in terms of their absolute relative bias (ARBias) and relative root mean square error (RRMSE), where,

$$ARBias(\hat{\gamma}) = \left|\frac{\hat{\gamma}-\gamma}{\gamma}\right| \qquad and \qquad RRMSE(\hat{\gamma}) = \frac{\sqrt{MSE(\hat{\gamma}-\gamma)}}{\gamma}$$

obtained, the results are given in the following table.

Table 1: The ML estimate, ARBias, RRMSE values of parameter γ

M		$\gamma = 0.3$	$\gamma = 0.5$	$\gamma = 0.6$
30	$\hat{\theta}$	0.302794	0.505582	0.605575
	ARBias	0.00931498	0.0111636	0.0092915
	RRMSE	0.107986	0.0985052	0.0974678
50	$\hat{\theta}$	0.302424	0.501672	0.603468
	ARBias	0.00808135	0.0033446	0.00578006
	RRMSE	0.0833185	0.0753077	0.078267
100	$\hat{\theta}$	0.301293	0.500336	0.601472
	ARBias	0.00430938	0.00067187	0.0024534
	RRMSE	0.0595189	0.0565896	0.0536481

Table. 1 shows the results of maximum likelihood estimate for the parameter γ:

1. The ML estimates decrease as sample size increases.
2. In the context of computational complexities, MLE includes solving non-linear equations and they need calculated by some numerical methods.
3. The ARBias and RRMSE of the estimated are decreased when sample size increases then, the MLE is a good estimator of γ.

4. APPLICATIONS

In this section, we provide renewable energy data in Saudi Arabia to illustrate the flexibility and importance of the inverse Ishita distribution. To assess the fit of the inverse Ishita distribution, we will compare it with two well-known competitive models, which are: two-parameter Weibull distribution and the Shanker distribution was introduced by [8]. The PDFs of these models are as follows. The PDF of Weibull distribution is:

$$f(x) = \frac{\beta}{\gamma}\left(\frac{x}{\gamma}\right)^{\beta-1} e^{-(x/\gamma)^{\beta}} \quad , x > 0, \gamma, \beta > 0.$$

Also, the PDF of the Shanker distribution is

$$f(x) = \frac{\gamma^2}{\gamma^2+1}(\gamma + x)e^{-\gamma x} \quad ,x > 0, \gamma > 0.$$

Additionally, we employ goodness of fit tests to compare the models such as:

AIC (Akaike information criterion) $= -2l(\hat{\gamma}) + 2q$,

BIC (Bayesian information criterion) $= -2l(\hat{\gamma}) + q\log(m)$,

$CAIC$ (consistent Akaike information criterion) $= -2l(\hat{\gamma}) + \frac{2qm}{m-q-1}$

and

$HQIC$ (Hannan − Quinn information criterion)
$= -2l(\hat{\gamma}) + 2q\, log(\log(m))$.

where $l(\hat{\gamma})$ denotes the log likelihood function, m is the sample size, and q represents the number of parameters. About other information for these measures see [9]. The smallest value of these measures determines the model that better fits the data.
In the following, we considered the data:

4.1 Data of average daily radiation in Saudi Arabia

Providing a wide variety of energy sources will enhance the economic performance of the Kingdom of Saudi Arabia. Reliance on renewable energy such as solar, wind and tidal energy will have a positive impact on the environment as it will reduce harmful emissions generated by conventional power plants. The kingdom of Saudi Arabia is characterized by the abundance of solar energy sources and vast lands. It is also one of the countries with the highest rates of solar radiation in the world. Therefore, in this study we will discuss data of the average daily radiation in (wh /m2/day) in Saudi Arabia from 2013 to 2022.
There are three types of radiation: global horizontal, direct normal and diffuse horizontal radiations, where the global horizontal is defined as the solar panels allow the horizontal radiation (GHI) to be converted directly into electrical energy using an inverter to convert solar energy into alternating current that is compatible with the grid, while the direct normal is defined as the use of reflective systems to direct solar radiation to a receiving device usually located at the top of a solar tower it converts concentrating solar power into thermal energy, on other hand the diffuse horizontal is defined as they are solar panels that allow the conversion of diffuse horizontal radiation (DHI) direct into electrical energy.

4.2 Data of global horizontal irradiance

These data of average daily of global horizontal radiation (Wh/m^2/day)
5585, 5725, 6042, 6107, 5577, 6333, 5916, 5533, 5759, 5651.

Table 2: The ML estimates of model's parameters and the goodness of fit measures for the global horizontal irradiance from 2013 to 2022.

Model	**ML estimates**		**Statistics**			
	$\hat{\gamma}$	$\hat{\beta}$	**AIC**	**BIC**	**HQIC**	**CAIC**
Inverse Ishita	5812.07	-	195.39	195.693	195.058	195.89
Weibull	2457.44	0.928826	207.405	208.01	206.741	209.119
Shanker	0.000343477	-	201.546	201.849	201.214	202.046

4.3 Data of direct normal irradiance

These data of average daily of direct normal radiation (Wh/m^2/day):
6137, 5911, 5401, 5884, 5242, 5377, 5559, 5727, 5795, 5375.

Table 3: The ML estimates of model's parameters and the goodness of fit measures for the direct normal irradiance data from 2013 to 2022.

Model	**ML estimates**		**Statistics**			
	$\hat{\gamma}$	$\hat{\beta}$	AIC	BIC	HQIC	CAIC
Inverse Ishita	5627.21	-	194.775	195.058	194.424	195.255
Weibull	2302.6	0.868651	207.552	208.157	206.888	209.266
Shanker	0.00035456	-	200.917	201.219	200.585	201.417

4.4 Data of diffuse horizontal irradiance

These data of average daily of diffuse horizontal radiation (Wh/m^2/day):
1751, 1944, 2383, 2152, 2122, 2487, 2153, 1914, 1958, 2181.

Table 4: The ML estimates of model's parameters and the goodness of fit measures for the direct normal irradiance data from 2013 to 2022.

Model	**ML estimates**		**Statistics**			
	$\hat{\gamma}$	$\hat{\beta}$	AIC	BIC	HQIC	CAIC
Inverse Ishita	2083.62	-	175.037	175.339	174.705	175.537
Weibull	861.198	0.897796	187.727	188.332	187.063	189.441
Shanker	0.000950344	-	181.273	181.576	180.942	181.773

In Tables 2-4, the values of AIC, BIC, HQIC and CAIC for the inverse Ishita distribution are the smallest compared to other distributions. The HQIQ was also the least value among the measures, which makes it the most suitable for this data. It is clear from the results that our proposed distribution is a better fit model for the renewable energy data in Saudi Arabia compared to Weibull and Shanker distributions.

5. CONCLUSION

In this article, we discussed the goodness of fit tests for the inverse Ishita distribution with one parameter. Further, the maximum likelihood estimator was derived to estimate the unknown parameter of the distribution. Also, numerical simulation was used to verify the behavior of maximum likelihood at different complete sample sizes and different values of parameter. Finally, to illustrate the usefulness and flexibility of the distribution it was applied to renewable energy data in Saudi Arabia and compared to specific models. According to goodness of fit tests, the inverse Ishita distribution shows a better fit for renewable energy data than other existing models.

Conflicts of Interest

No conflict of interest.

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